



## Dynamics of $N$ -person snowdrift games in structured populations

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### HIGHLIGHTS

- ▶ Achieving a common goal requires the cooperation of at least some individuals.
- ▶ This can be modeled by an  $N$ -person snowdrift game, a variant of Public Goods Games.
- ▶ Social networks influence the emerging behavioral dynamics.
- ▶ Homogeneous networks help the coordination towards stable levels of cooperation.
- ▶ Heterogeneous networks create a new scenario with multiple behavioral equilibria.

### GRAPHICAL ABSTRACT

When a snowdrift traps several individuals, work is required to shovel off the snow and let everyone get home, an effort which is shared among those who participate. This situation provides a powerful metaphor one encounters in many real life situations, in which the completion of a task by a group toward achieving a common goal requires the cooperation of at least some of its members, who share the workload. The  $N$ -person snowdrift game is a Public Goods Game that conveniently models such cases. Here, each individual has the option to help (or not) shoveling the snow, such that the more the individuals who shovel, the less the effort each one has to invest in order to surpass (in the illustrated example) the blocked railway. On the other hand, once the snow is removed, everyone will be able to reach his or her destination (and, game-wise, everyone gets the same benefit). In this manuscript, we study how a social network affects the behavioral dynamics under such class of dilemmas. We show that homogeneous social structures enhance the chances of coordinating toward stable levels of cooperation, while heterogeneous network structures create multiple internal equilibria, departing significantly from the reference scenario of a structureless population (artwork by António Araújo, 2012).



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### ABSTRACT

In many real-life situations, the completion of a task by a group toward achieving a common goal requires the cooperation of at least some of its members, who share the required workload. Such cases are conveniently modeled by the  $N$ -person snowdrift game, an example of a Public Goods Game. Here we study how an underlying network of contacts affects the evolutionary dynamics of collective action modeled in terms of such a Public Goods Game. We analyze the impact of different types of networks in the global, population-wide dynamics of cooperators and defectors. We show that homogeneous social structures enhance the chances of coordinating toward stable levels of cooperation, while heterogeneous network structures create multiple internal equilibria, departing significantly from the reference scenario of a well-mixed, structureless population.

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## 1. Introduction

*Evolutionary Game Theory* (EGT) plays a central role in the study of the emergence and evolution of cooperation at all scales, a problem that has received increasing attention (Axelrod and Hamilton, 1981; Maynard Smith, 1982; Macy and Flache, 2002; Skyrms, 2004; Nowak, 2006a, 2006b; Sigmund, 2009). Traditionally, interactions have been modeled in terms of one-shot, symmetric two-person dilemmas of Cooperation such as the *Prisoner's Dilemma* (PD) (Rapoport and Chammah, 1965; Axelrod and Hamilton, 1981), the *Snowdrift Game* (SG) (Sugden, 1986), or the *Stag-Hunt Game* (SH) (Skyrms, 2004). However, many real-life situations involve decisions derived from groups composed of more than two individuals. This type of collective action problems—which abound, not only in humans (Hardin, 1968; Ostrom, 1990; Sigmund, 2009; Potete et al., 2010) but also in other upper primates (Boyd and Richerson, 1988; Kollock, 1998)—is best described in the framework of  $N$ -person games (Hardin, 1968; Dawes, 1980; Boyd and Richerson, 1988; Kollock, 1998; Hauert et al., 2004, 2007; Santos et al., 2008; Gokhale and Traulsen, 2010; Santos and Pacheco, 2011; Van Segbroeck et al., 2012), in particular Public Goods Games.

Here we investigate the  $N$ -person generalization of the SG (NSG) (Sugden, 1986; Zheng et al., 2007; Souza et al., 2009), a powerful metaphor for modeling situations in which performing a task which confers the same benefit to the entire group depends on the cooperation of several of its members, who share the required workload. A classic example that illustrates this dilemma assumes  $N$  travelers trapped in a train blocked by a snowdrift, as illustrated in Fig. 1. Each individual can choose whether or not to cooperate by shoveling the snow. Those who cooperate divide the workload, while all collect the benefit of resuming their journey home. Those who do not cooperate may enjoy the sun on the train's roof or use the time to take a picture

(see Fig. 1), while the others do the shoveling. When the snow is removed, everyone will manage to get to the train destination. It may also happen that the benefit is obtained only when a minimum threshold of individuals cooperates (Pacheco et al., 2009; Souza et al., 2009; Santos and Pacheco, 2011): in line with the previous example, the timely removal of the snow may require the combination of efforts from several individuals.

There are several examples in everyday life that conform to the NSG. When lion(esse)s gather to hunt a prey (Stander, 1992), sometimes not all of them participate in the hunting, yet, all have access to the same benefit. Whether the benefit is (equally) shared among all group members amounts to a redefinition of what is the benefit. Or take for instance the construction of a church by a group of colonizers who arrived to some newfound land. The effort of a single individual is not enough to build it up, and the more the individuals who contribute, the less the effort each one has to invest. In the end, the benefit of having such building is independent of how many participate in the enterprise.

In an infinite *well-mixed* (WM) population, where all individuals have equal chances to interact with everyone else, the evolutionary dynamics is conveniently described, in the framework of EGT, by the replicator equation (Taylor and Jonker, 1978; Hofbauer and Sigmund, 1998), where individual fitness is associated with the game payoff. Similarly to its two-person counterpart, the evolution of a population of cooperators ( $C$ s, those who contribute) and defectors ( $D$ s, those who refuse to do so yet reap the benefit) under a NSG leads the population toward a configuration in which one observes a coexistence of the two strategies, a result that persists in finite populations (Zheng et al., 2007; Souza et al., 2009). However, very often such a WM assumption is not realistic: indeed, one expects that different individuals interact with different (albeit small) sub-sets of the whole population. Such topological constraints can be conveniently described by



**Fig. 1.** The  $N$ -person Snowdrift Game (NSG) metaphor: In the NSG one assumes  $N$  individuals trapped by a snowdrift. Each individual has the option to help (or not) shoveling the snow, such that the more the individuals who shovel, the less the effort each one has to invest in order to surpass (in the illustrated example) the blocked railway. On the other hand, once the snow is removed, all will be able to reach their destination (and, game-wise, everyone gets the same benefit). Moreover, as is often the case in collective dilemmas, the benefit resulting from resuming the trip may be obtained only whenever a minimum number of individuals decide to cooperate (artwork by António Araújo, 2012).

means of complex networks, in which individuals are assigned to the nodes of the network whereas links between them define their interconnectedness and pattern of social interactions. As illustrated in Fig. 2, each neighborhood may characterize a  $N$ -person game in which an individual engages (Szabó and Hauert, 2002; Santos et al., 2008; Wu et al., 2009; Ji et al., 2010; Lei et al., 2010; Szolnoki and Perc, 2010a, 2010b; Ji et al., 2011; Szolnoki et al., 2011; Vukov et al., 2011).

Let us start by summarizing the main results in infinite and finite WM populations before investigating in which way the problem of collective action formulated in terms of the NSG is affected by the underlying networks that structuralize the population.

## 2. $N$ -person snowdrift game in well-mixed populations

We consider an infinite WM population of individuals behaving either as unconditional cooperators (Cs) or defectors (Ds). The threshold  $M$  ( $1 \leq M \leq N$ ) defines a minimum number of Cs in the group required to obtain the collective benefit, so as to encompass

several situations in which the contribution of a single individual ( $M=1$ ) is not enough to achieve the common goal. The payoffs of Cs and Ds can hence be written as  $\Pi_C(n_C) = H(n_C - M)(c(1/M - 1/n_C) + b) - c/M$  and  $\Pi_D(n_C) = H(n_C - M)b$  respectively, where  $n_C$  is the number of Cs in the group,  $c$  is the total cost involved in achieving  $b$ , the common benefit obtained by each individual of the group, regardless of her strategy, when  $n_C \geq M$  (throughout all this manuscript, we shall take  $b=1$ ) and  $H(x-a)$  is the Heaviside step function which is 1 whenever  $x \geq a$ , and 0 otherwise.

Random sampling of groups yields the following average fitness of Cs ( $f_C$ ) and Ds ( $f_D$ ) (Hauert et al., 2006; Pacheco et al., 2009; Souza et al., 2009):

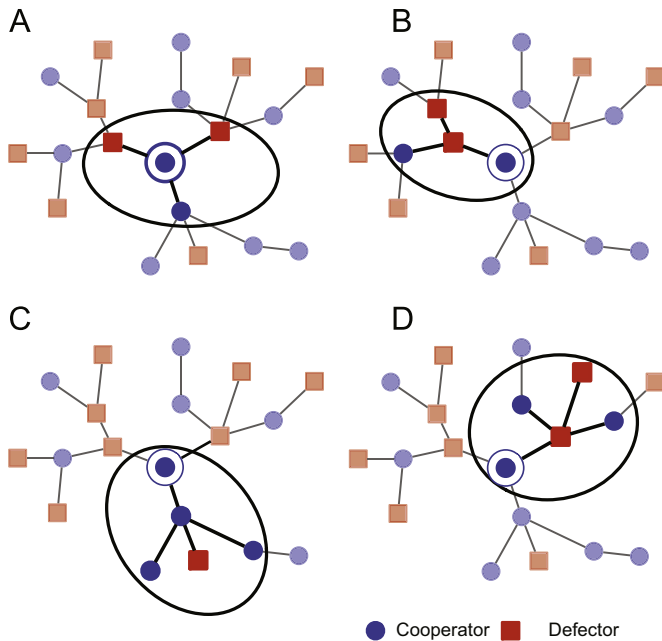
$$f_C(x) = \sum_{n_C=0}^{N-1} \binom{N-1}{n_C} x^{n_C} (1-x)^{N-1-n_C} \Pi_C(n_C+1) \quad (1)$$

and

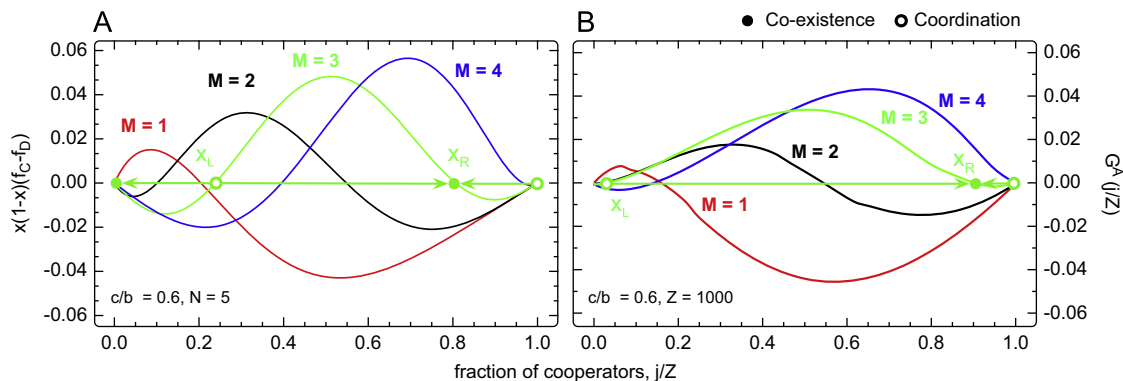
$$f_D(x) = \sum_{n_C=0}^{N-1} \binom{N-1}{n_C} x^{n_C} (1-x)^{N-1-n_C} \Pi_D(n_C) \quad (2)$$

Denoting, as usual, by  $x(1-x)$  the fraction of Cs (Ds) in the population, the replicator equation  $\dot{x} = x(1-x)(f_C(x) - f_D(x))$  allows us to follow the time evolution of Cs in the population. In particular, the sign of the so-called gradient of selection, here denoted by  $g(x)$  ( $g(x) \equiv \dot{x}$ ), indicates which strategy increases in abundance. Fig. 3A shows  $g(x)$  for different threshold levels: for  $M=1$ ,  $g(x)$  dictates a single stable equilibrium, which can be computed numerically for an arbitrary  $N$ , while for  $M > 1$  a richer evolutionary dynamics emerges, typically characterized by two interior roots of  $g(x)$  one unstable at lower frequency of Cs and another stable, at a higher frequency of Cs. This dynamics characterized by two basins of attraction typically vanishes for a critical  $c/b$  (Souza et al., 2009), above which a single stable fixed point remains for  $x=0$ , similar to the  $N$ -person prisoner's dilemma.

For finite populations of size  $Z$ , the general dynamical picture remains the same, as long as  $Z \gg N$  (Souza et al., 2009). Here, the binomial sampling is replaced by a hyper-geometric sampling. Evolution in finite populations can be conveniently studied by means of a birth–death (BD) process, combined with the pairwise comparison rule (Szabó and Tóke, 1998; Traulsen et al., 2006, 2007). In this case, and in each step of the BD process, an individual  $A$  adopts the strategy of a randomly selected neighbor  $B$  with a probability given by the Fermi distribution  $p = [1 + e^{-\beta(f_B - f_A)}]^{-1}$ , where  $\beta$  stands for the intensity of selection regulating the accuracy of the imitation process. As a result, evolution proceeds as a balance between the probabilities to increase ( $T^+$ ) and decrease ( $T^-$ ) the number of Cs in the



**Fig. 2.** Groups on networks: the focal individual (largest blue disk) participates in the games centered on her neighbors (groups B–D) as well as the game centered on herself (group A). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 3.** Gradient of selection for the  $N$ -person snowdrift game. We fixed  $c/b=0.6$ ,  $N=5$ ,  $\beta=1.0$  and plot, for different values of  $M$  (A) the gradient of selection  $g(x)$  in infinite, well-mixed populations and (B) the average gradient of selection  $G^A(j/Z)$  in finite populations structured along the links of HRND networks (see main text for details).

population, which can be written as

$$T^\pm(j) = \frac{j}{Z} \frac{Z-j}{Z-1} [1 + e^{\mp \beta(f_C(j) - f_D(j))}]^{-1} \quad (3)$$

where  $j$  stands for the total number of Cs in the population. The finite population equivalent of the gradient of selection  $g(x)$  can be written as

$$G(j) \equiv T^+(j) - T^-(j) = \frac{j}{Z} \frac{Z-j}{Z-1} \tanh \left[ \frac{\beta}{2} (f_C(j) - f_D(j)) \right] \quad (4)$$

which exhibits similar roots as those of  $g(x)$  for  $Z \gg N$  (Souza et al., 2009).

### 3. Evolutionary dynamics in structured populations

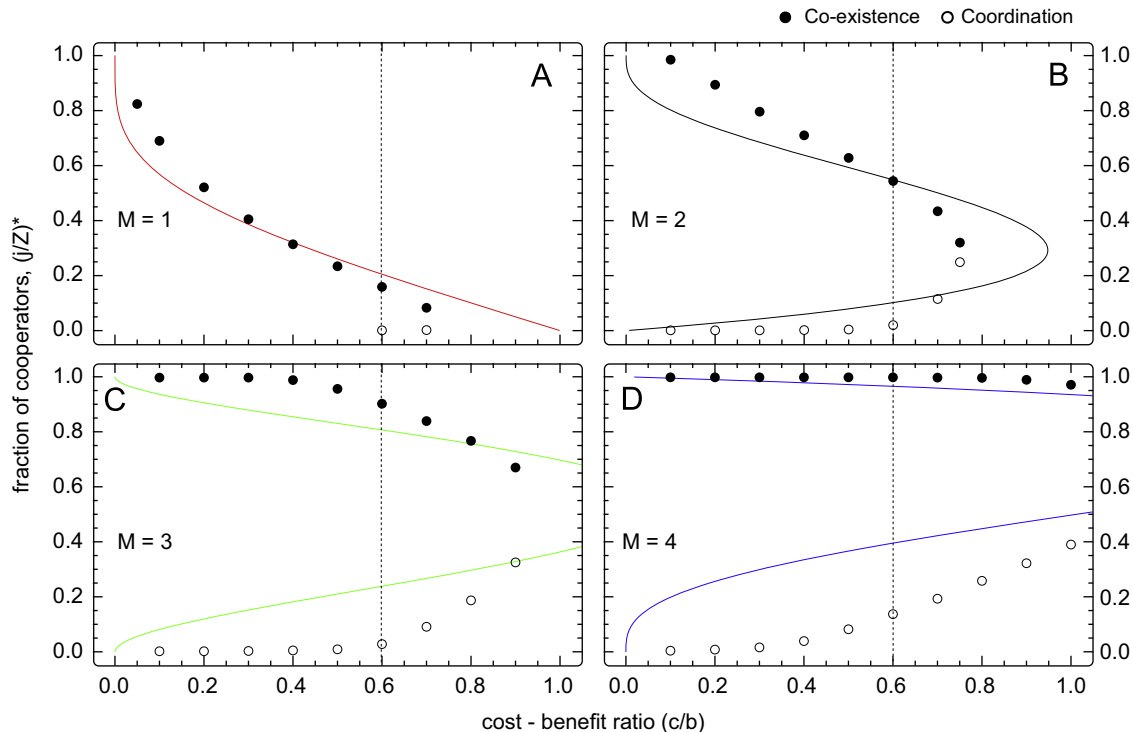
In line with previous studies (Santos et al., 2008; Vukov et al., 2011), an individual's fitness is determined by the payoffs resulting from the game centered on herself plus the  $k$  games centered on her  $k$  neighbors, as depicted in Fig. 2. In homogeneous populations every individual takes part in the same number of games, all with the same size ( $k+1$  in both cases). However, this scenario does not take into account an important feature of social networks: its diversity (Barabási and Albert, 1999; Dorogovtsev and Mendes, 2003; Santos et al., 2012). Often individuals face different numbers of collective dilemmas (depending on their social position) that may also have different sizes. Such levels of social diversity can be modeled by considering a heterogeneous network of interactions as pictured in Fig. 2. In the following section we compare the evolutionary dynamics on both homogeneous and heterogeneous networks. To this end, we consider homogeneous random networks (HRND) (Santos et al., 2005) for the former, and scale-free networks generated with the Barabási–Albert algorithm (BA) of growth and preferential attachment (Barabási and Albert, 1999) as representative of the latter.

We consider populations of size  $Z=10^3$  with an average connectivity ( $\langle k \rangle$ ) of 4. We maintain the strategy updating via the pairwise comparison rule (already described in Section 2), in which individuals now randomly choose a neighbor to be imitated. We resort to computer simulations to assess the evolutionary outcome of such populations: in particular, we compute the so-called *average gradient of selection* (AGoS) (Pinheiro et al., 2012a, 2012b), a quantity which provides information equivalent to that given by  $G(k)$  in Eq. (4) but which, now, explicitly takes into consideration network structure effects. In short, and for each individual  $i$ , we compute the probability that this individual changes behavior at time  $t$ ,  $T_i(t) = k_i^{-1} \sum_{m=1}^{n_i} [1 + e^{-\beta(f_i(t))}]^{-1}$ , where  $k_i$  stands for the degree of node  $i$  and  $n_i$  for the number of neighbors of  $i$  having a different strategy. At a given time  $t$  of simulation  $p$  we define  $G_p(j, t) = T_p^+(j, t) - T_p^-(j, t)$  (with

$T_p^\pm(j, t) = Z^{-1} \sum_{i=1}^{AllDs} T_i(t)$  for a state with  $j$  Cs in a population of size  $Z$ . The time-dependent AGoS at generation  $t_g$  (1 generation means  $Z$  iterations),  $G^A(j, t_g)$ , is then computed by averaging over the last  $Z$  time-steps, that is,  $G^A(j, t_g) = c_j(t_g)^{-1} \sum_{t=t_g-1}^{t_g} \sum_{p=1}^{\Omega} G_p(j, t)$ , where  $c_j(t_g)$  accounts for the number of times the population was observed in state  $j$  during generation  $t_g$ . For a given network type, we run  $\Omega = 2.5 \times 10^7$  simulations (using  $10^2$  networks of each type) starting from random initial conditions.

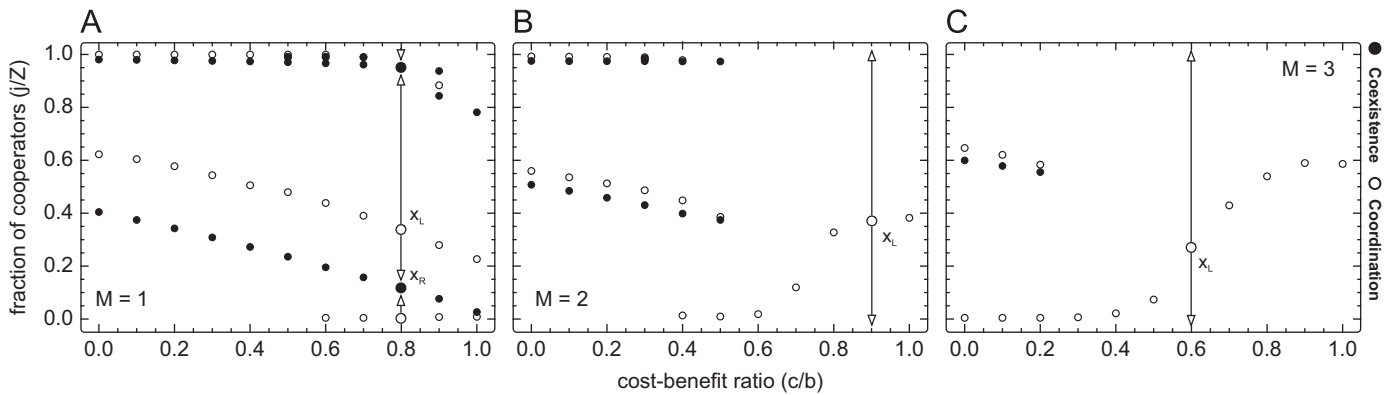
### 4. Results and discussion

Fig. 3A and B shows typical profiles of the gradients of selection, both in well-mixed and HRND populations, respectively. Comparing the two panels, we observe that while the nature of the dilemma remains unchanged, (i) HRND networks facilitate coordination: regardless of the value of  $M$ , the coordination point ( $x_L$ ) is shifted to the left (requiring less Cs) when



**Fig. 4.** Location of the interior fixed points of the AGoS at the 75th generation for HRND populations (open and solid circles) as a function of  $c/b$  and for  $1 \leq M \leq 4$ . The corresponding results for well-mixed populations are drawn with solid lines for comparison. Vertical dashed lines indicate the values used in Fig. 3. Whenever these vertical lines cross the solid curves in two points, the upper root corresponds to a co-existence equilibrium, whereas the lower one to a coordination equilibrium.





**Fig. 5.** Evolutionary dynamics on BA populations. Position of the internal roots of  $G^A(j/Z, g)$  at the 150th generation is drawn as a function of  $c/b$  for (A)  $M=1$ , (B)  $M=2$  and (C)  $M=3$ . Solid (open) circles indicate a co-existence (coordination) point ( $\beta=1.0$ ).

compared with its position in well-mixed populations. Furthermore, (ii) whenever individuals face stringent requirements to meet goals (large  $M/N$ ) the coexistence point ( $x_R$ ) is shifted to the right (coexistence involving more Cs). In both cases we obtain a more favorable scenario for cooperation in HRND structured populations compared to well-mixed populations. It is also noteworthy that, for large  $M/N$ , the relative size of the two basins of attraction combined with stochastic effects renders full cooperation as the most prevalent state, whereas for low  $M/N$ , the population will remain in the vicinity of the co-existence point ( $x_R$ ) most of the time. Finally, both in well-mixed and HRND-structured populations, large values of  $c/b$  (e.g.,  $c/b > 0.8$ ) slightly shift  $x_R$  to lower values of the fraction  $j/N$  of Cs.

The results in Fig. 3 allow the visualization of the gradients of selection for specific values of  $c/b$ . Differently, in Fig. 4 we provide an overview of the evolutionary dynamics for the full range of  $c/b$ , by drawing the location of the interior roots of the AGoS for the different values of  $M$  (circles), in comparison with the well-mixed case (solid lines), showing how HRND networks favor cooperation by reducing the fraction of cooperators ( $x_L$ ) needed to escape full defection ( $j/Z=0$ ). Furthermore, for low values of  $c/b$ , co-existence ( $x_R$ ) occurs for larger fractions of cooperators, a scenario which is overturned for large values of  $c/b$ , in which cooperation may be hindered, as in two-person SG (Hauert and Doebeli, 2004).

Homogeneous networks are, however, not regarded as good examples of realistic social structures. Instead, these exhibit a marked degree of heterogeneity (in practice, a large number of individuals with few links whereas a few have a large number of links) combined with small-world effects, characteristics that are both present, albeit at extreme levels, on BA networks.

Fig. 5 provides, similar to Fig. 4, but now for BA populations, the location of the internal roots of  $G^A(j, t_g)$  as a function of  $c/b$ . We consider the cases  $M=1$  to  $M=3$ , the latter corresponding to imposing unanimity of cooperation for the smallest groups of the population (for our particular choice of the average connectivity  $\langle k \rangle$ , for which we used  $\langle k \rangle = 4$ ). The intrinsically dual nature (co-existence and coordination) of the NSG creates a multitude of internal roots which result from the occurrence of evolutionary deadlocks associated with particular motifs of the network (Gómez-Gardeñes et al., 2008; Pinheiro et al., 2012a), reflected in the appearance of quasi-stationary states close to full cooperation and full defection. This happens, in particular, for the case of strong selection, and can be clearly identified in panels A and B of Fig. 5. For this reason, and contrary to the case of HRND networks, the AGoS in BA networks do not converge rapidly into a stationary state, due to the continuous invasion and counter-invasion induced by highly connected nodes. Analysis of domain growing processes in spatially structured populations (Szabó and Fáth,

2007) shows that the appearance, simultaneously, of more than one stable equilibrium often leads to a competition between them, resulting in the emergence of a dominant configuration. This process, which has been investigated in detail in spatial lattices, also in ecological contexts (Szabó et al., 1999), is largely unexplored in networks exhibiting heterogeneity and small-world structures, and it would be interesting to understand in detail how competition is affected by network properties such as size, cluster coefficient or average path length. In what concerns the population (network) size  $Z$ , we find that the number of competing equilibria remains unaltered if we increase  $Z$  by one order of magnitude. Moreover, computation of the quasi-stationary distributions for  $Z=10^3$ , that is, the prevalence of each state before fixation, taking into account the first  $2.5 \times 10^3$  generations shows no trace of any disappearance of these edge coexistence states, a feature which seems to rely on the robustness of the coordination root  $x_L$ .

As we increase the coordination requirements needed to achieve a collective benefit ( $M > 1$ , see Fig. 5B and C), local co-existences will become harder to achieve which, combined with harsher dilemmas (large  $c/b$ ), in the long run lead the population-wide dynamics toward a coordination dilemma (Skyrms, 2004), with a single unstable root and two stable monomorphic states. For large values of  $M$  and  $c/b$ , it is noteworthy that global coordination can be reached for values of  $c/b$  in which widespread defection would pervade in well-mixed populations (i.e.  $M=1$  and  $c/b=1.0$ ), showing how heterogeneous networks may transform a defection dominance dilemma into a coordination problem.

## 5. Conclusion

The present study puts in evidence the impact of structured populations in the dynamics of cooperative collective action associated with the NSG. Due to its twofold nature of coordination and coexistence (Souza et al., 2009) — in well-mixed populations it exhibits a pair of coordination and coexistence points when a threshold  $M$  is introduced — as such, the NSG surpasses in complexity most dilemmas previously studied in structured populations (Szabó and Fáth, 2007; Perc and Szolnoki, 2010; Santos et al., 2012).

In order to handle such a degree of complexity, we have explicitly addressed the global dynamics (Pinheiro et al., 2012a) created by each network structure, enabling a direct comparison with the results previously obtained in unstructured populations (Zheng et al., 2007; Souza et al., 2009). We show that evolution on homogeneous networks exhibits a population-wide behavior qualitatively similar to that observed in well-mixed populations.

Notwithstanding, homogeneous social structures are able to reduce the efforts needed to achieve a stable fraction of cooperators which, in turn, increases with increasing  $M$ . Heterogeneous networks, on the other hand, lead to more complex evolutionary dynamics scenarios, with a multitude of internal equilibria, whereas for high values of  $c/b$  and  $M > 1$ , the increasing difficulty to locally coordinate actions transforms the overall population-wide dynamics into a coordination problem.

Finally, our results support the idea that stringent requirements to achieve a collective benefit significantly raise the chances of cooperation, thereby escaping the tragedy of the commons.

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## References

- Axelrod, R., Hamilton, W.D., 1981. The evolution of cooperation. *Science* 211, 1390–1396.
- Barabási, A.L., Albert, R., 1999. Emergence of scaling in random networks. *Science* 286, 509–512.
- Boyd, R., Richerson, P.J., 1988. The evolution of reciprocity in sizable groups. *J. Theor. Biol.* 132, 337–356.
- Dawes, R.M., 1980. Social dilemmas. *Annu. Rev. Psychol.* 31, 169–193.
- Dorogovtsev, S.N., Mendes, J.F.F., 2003. *Evolution of Networks: From Biological Nets to the Internet and WWW*. Oxford University Press, USA, Oxford.
- Gokhale, C.S., Traulsen, A., 2010. Evolutionary games in the multiverse. *Proc. Natl. Acad. Sci. USA* 107, 5500.
- Gómez-Gardeñes, J., Ponceña, J., Mario Flórida, L., Moreno, Y., 2008. Natural selection of cooperation and degree hierarchy in heterogeneous populations. *J. Theor. Biol.* 253, 296–301.
- Hardin, G., 1968. The tragedy of the commons. *Science* 162, 1243–1248.
- Hauert, C., Doebeli, M., 2004. Spatial structure often inhibits the evolution of cooperation in the snowdrift game. *Nature* 428, 643–646.
- Hauert, C., Haiden, N., Sigmund, K., 2004. The dynamics of public goods. *Discr. Cont. Dynam. Syst. Ser. B* 4, 575–588.
- Hauert, C., Michor, F., Nowak, M.A., Doebeli, M., 2006. Synergy and discounting of cooperation in social dilemmas. *J. Theor. Biol.* 239, 195–202.
- Hauert, C., Traulsen, A., Brandt, H., Nowak, M.A., Sigmund, K., 2007. Via freedom to coercion: the emergence of costly punishment. *Science* 316, 1905–1907.
- Hofbauer, J., Sigmund, K., 1998. *Evolutionary Games and Population Dynamics*. Cambridge University Press, Cambridge.
- Ji, M., Xu, C., Hui, P., 2011. Effects of dynamical grouping on cooperation in  $N$ -person evolutionary snowdrift game. *Phys. Rev. E* 84, 036113.
- Ji, M., Xu, C., Zheng, D.F., Hui, P., 2010. Enhanced cooperation and harmonious population in an evolutionary  $N$ -person snowdrift game. *Physica A* 389, 1071–1076.
- Kollock, P., 1998. Social dilemmas: the anatomy of cooperation. *Annu. Rev. Sociol.* 183–214.
- Lei, C., Wu, T., Jia, J.Y., Cong, R., Wang, L., 2010. Heterogeneity of allocation promotes cooperation in public goods games. *Physica A* 389, 4708–4714.
- Macy, M.W., Flache, A., 2002. Learning dynamics in social dilemmas. *Proc. Natl. Acad. Sci. USA* 99, 7229–7236.
- Maynard Smith, J., 1982. *Evolution and the Theory of Games*. Cambridge University Press.
- Nowak, M.A., 2006a. *Evolutionary Dynamics: Exploring the Equations of Life*. Belknap Press.
- Nowak, M.A., 2006b. Five rules for the evolution of cooperation. *Science* 314, 1560–1563.
- Ostrom, E., 1990. *Governing the Commons: The Evolution of Institutions for Collective Action*. Cambridge University Press.
- Pacheco, J.M., Santos, F.C., Souza, M.O., Skyrms, B., 2009. Evolutionary dynamics of collective action in  $N$ -person stag hunt dilemmas. *Proc. R. Soc. B: Biol. Sci.* 276, 315–321.
- Perc, M., Szolnoki, A., 2010. Coevolutionary games—a mini review. *BioSystems* 99, 109–125.
- Pinheiro, F.L., Pacheco, J.M., Santos, F.C., 2012a. From local to global dilemmas in social networks. *PLoS One* 7, e32114.
- Pinheiro, F.L., Santos, F.C., Pacheco, J.M., 2012b. How selection pressure changes the nature of social dilemmas in structured populations. *New J. Phys.* 14, 073035.
- Poteete, A.R., Janssen, M.A., Ostrom, E., 2010. *Working Together: Collective Action, the Commons, and Multiple Methods in Practice*. Princeton University Press.
- Rapoport, A., Chammah, A.M., 1965. *Prisoner's Dilemma: A Study in Conflict and Cooperation*. University of Michigan Press.
- Santos, F., Rodrigues, J., Pacheco, J., 2005. Epidemic spreading and cooperation dynamics on homogeneous small-world networks. *Phys. Rev. E* 72, 056128.
- Santos, F.C., Pacheco, J.M., 2011. Risk of collective failure provides an escape from the tragedy of the commons. *Proc. Natl. Acad. Sci. USA* 108, 10421.
- Santos, F.C., Santos, M.D., Pacheco, J.M., 2008. Social diversity promotes the emergence of cooperation in public goods games. *Nature* 454, 213–216.
- Santos, F.C., Pinheiro, F.L., Lenaerts, T., Pacheco, J.M., 2012. The role of diversity in the evolution of cooperation. *J. Theor. Biol.* 299, 88–96.
- Sigmund, K., 2009. *The Calculus of Selfishness*. Princeton University Press.
- Skyrms, B., 2004. *The Stag Hunt and the Evolution of Social Structure*. Cambridge University Press.
- Souza, M.O., Pacheco, J.M., Santos, F.C., 2009. Evolution of cooperation under  $N$ -person snowdrift games. *J. Theor. Biol.* 260, 581–588.
- Stander, P., 1992. Cooperative hunting in lions: the role of the individual. *Behav. Ecol. Sociobiol.* 29, 445–454.
- Sugden, R., 1986. *The Economics of Rights, Co-operation and Welfare*. Blackwell Oxford.
- Szabó, G., Tóke, C., 1998. Evolutionary prisoner's dilemma game on a square lattice. *Phys. Rev. E* 58, 69.
- Szabó, G., Hauert, C., 2002. Phase transitions and volunteering in spatial public goods games. *Phys. Rev. Lett.* 89, 118101.
- Szabó, G., Fáth, G., 2007. Evolutionary games on graphs. *Phys. Rep.* 446, 97–216.
- Szabó, G., Santos, M.A., Mendes, J.F.F., 1999. Vortex dynamics in a three-state model under cyclic dominance. *Phys. Rev. E* 60, 3776.
- Szolnoki, A., Perc, M., 2010a. Reward and cooperation in the spatial public goods game. *Europhys. Lett.* 92, 38003.
- Szolnoki, A., Perc, M., 2010b. Impact of critical mass on the evolution of cooperation in spatial public goods games. *Phys. Rev. E* 81, 057101.
- Szolnoki, A., Szabo, G., Perc, M., 2011. Phase diagrams for the spatial public goods game with pool-punishment. *Phys. Rev. E* 83, 036101.
- Taylor, P.D., Jonker, L.B., 1978. Evolutionary stable strategies and game dynamics. *Math. Biosci.* 40, 145–156.
- Traulsen, A., Nowak, M.A., Pacheco, J.M., 2006. Stochastic dynamics of invasion and fixation. *Phys. Rev. E* 74, 011909.
- Traulsen, A., Pacheco, J.M., Nowak, M.A., 2007. Pairwise comparison and selection temperature in evolutionary game dynamics. *J. Theor. Biol.* 246, 522–529.
- Van Segbroeck, S., Pacheco, J.M., Lenaerts, T., Santos, F.C., 2012. Emergence of fairness in repeated group interactions. *Phys. Rev. Lett.* 108, 158104.
- Vukov, J., Santos, F.C., Pacheco, J.M., 2011. Escaping the tragedy of the commons via directed investments. *J. Theor. Biol.* 287, 37–41.
- Wu, T., Fu, F., Wang, L., 2009. Partner selections in public goods games with constant group size. *Phys. Rev. E* 80, 026121.
- Zheng, D.F., Yin, H., Chan, C.H., Hui, P., 2007. Cooperative behavior in a model of evolutionary snowdrift games with  $N$ -person interactions. *Europhys. Lett.* 80, 18002.