

Managerial Flexibilities and Competition in Shopping Center Investment. Real Options Approach to Real Estate

Luciana Salles Barbosa^{*†}, Monika Tarsalewska^{‡§}, Alberto Sardinha^{¶||}

Abstract

Managerial flexibilities can add value to real estate investments. However, the competition can erode the value of management options. We propose a model which identifies the optimal time to launch the second stage of the project. We show how the probability of a competitor arrival can affect the investment threshold. Shopping center among all real estate investments is a specific example where the competition has particular importance as it affects the option to expand.

JEL Classification: C61, G13, G17, G31, R33

Keywords: Real Options, Shopping Centers, NPV, Competition

1. Introduction

Shopping Center investments are very sensitive to the economic situation of the country. Investments in this sector require capital-intensive outflows (mainly construction costs and

^{*}Instituto Superior Técnico and Instituto Superior de Economia e Gestão, UTL, Portugal

[†]E-mail: l40296@aln.iseg.utl.pt

[‡]Department of Accounting and Finance, Lancaster University Management School, UK

[§]E-mail: m.tarsalewska@lancaster.ac.uk

[¶]INESC-ID and Instituto Superior Técnico, UTL, Portugal

^{||}E-mail: jose.alberto.sardinha@ist.utl.pt

land cost), are associated with slow payback, and cannot be sold at short notice (unless a large price discount is given). There are also several uncertainties involved such as demand, rent/m², rent speed as well as the mix of stores. Furthermore, as emerging economies attract many investors, one has to take into account the effect of competition on the value of the project. Therefore, high sunk cost and low liquidity investment are very risky in dynamic economies where the investor faces significant competition.

The GDP growth in emerging markets over the last twenty years was several fold higher than in developed economies. Therefore, these economies are lucrative consumer markets attracting many investors. At the same time, these markets are characterized by high uncertainty over future profits. Economies with high degree of uncertainty, together with legislation and local government risks (authorizations, licenses, etc.) increase the volatility of the investment project.

The standard static valuation technique, such as NPV, underestimates the value of the project as the method does not take into account the flexibilities that are present in dynamic environments. For example, an option to expand when conditions are favorable or cut losses when the project is a failure. Real options theory provides a dynamic valuation technique that takes into account managerial flexibilities.

There is some empirical evidence that the real options are valued as a part of investment project in developed economies.¹ However, the real options analysis should be still applied

¹Several recent real option studies examine the effect of uncertainty on the real estate development. Capozza and Li (1994) show that land owners have the option to convert agricultural land into urban, and the optimal conversion rule depends on the distance to urban areas. Capozza and Gordon (1994) focus on how the density and capital intensity choices interact with the timing and value of residential or commercial developments. Quigg (1993) provides empirical evidence of the descriptive value of the option to wait based on actual real estate transactions in the US. Bulan, Mayer, and Somerville (2009) show empirically that the uncertainty delays investment but the competition decreases the value of the option in the region of US. Cunningham (2006) shows that real estate investors take into account the option value based on the data from US region (King County, Washington). Grovenstein, Kau, and Munneke (2011) use option approach to determine the value of development option and estimate the option premium in the region of Cook County, Illinois.

to project valuation in developing and emerging economies where the level of uncertainty is even higher. The purpose of this paper is therefore to present a model to value the shopping center investment where managerial flexibilities are valued and competition between shopping centers is taken into account. We show how to value the project with embedded options and the fear of preemption.

This article relates and contributes to the literature in several ways. Our first contribution is to the options literature, where we extend the Barone-Adesi and Whaley (1987) approximation. In particular, we show how introducing competition between investors changes the investment threshold.

The option valuation technique became very popular since developments of Merton (1973) and Black and Scholes (1973). Their closed-form formulas to value European options became widely used by practitioners. Finite time American options pose several problems as close form solutions do not exist and numerical approximation is time consuming. Barone-Adesi and Whaley (1987) approximation solves this problem and is now widely used as an effective method to value finite time American options. Further literature provided other methods of calculating the value of American options, however at a cost of computational efficiency.

Corporate finance literature saw the potential of using the techniques of option pricing to value investment projects. The term “real options” was coined by Myers (1977) and provides a methodology to better value investment projects in the presence of managerial flexibilities. The real options framework was introduced by Brennan and Schwartz (1985) to show an example of mineral extraction where future prices are uncertain. McDonald and Siegel (1986) derived the optimal investment rule which takes into account the value of waiting. They showed that the standard NPV valuation is grossly wrong.² Gryglewicz, Huisman, and Kort (2008) explores the implications of the timing of the project and option. In particular, when the project has finite life (but the option is infinite) the effect of uncertainty can accelerate the decision to invest. When the project is infinite and the option to invest in that project

²A detailed description of the different models and methodologies can be found in Dixit and Pindyck (1994) and Trigeorgis (1996). Also, Schwartz and Trigeorgis (2004) include classical readings where real options have been applied in several investment projects to account for the value of flexibility.

has limited durability then the standard results hold; however, the effect of uncertainty on the value of the option and exercise trigger depends on remaining option life.

Our second contribution is to apply the real option approach in the shopping center valuation. The early study of Titman (1985) uses the real option theory to estimate price of several empty lots in urban areas. He concludes that the potentiality of the lot is worth more than its immediate use in a construction, in the presence of uncertainties delaying investment. He argues that the existence of several empty lots, but of high sale price, in West Los Angeles was due to the presence of an option to wait, i.e., the future potential of the lot was more valuable than its immediate use for construction. Although his analysis provides some intuition it has many limiting assumptions, such as infinite time of the option and the cost of development is constant. Williams (1991) extends this study by introducing uncertainty over the cost of construction. He determines the optimal timing for development and abandonment of the property, as well as the optimal density in the presence of uncertainties about cost/m² and price/m². Yung Medeiros (2003) applies the real option valuation in a real estate market of an emerging market (Brazil). She extends Williams (1991) work by including taxes and a discount in the net cash inflow due to time spent in the construction process. Grenadier (1995) determines the inter-temporal optimal mix of tenants in shopping centers, where the landlord has both options to increase or decrease the current mix with exercise prices being the cost of mix adjustments. He shows that the difference between the dynamic and static strategies is the value added by the embedded options. Grenadier (1996) introduces the option game concepts to explain the behavior of real estate markets, linking the investment timing in strategic equilibrium to the boost or slowdown in development activity. Grenadier (2005) also presents a unified equilibrium approach to value real estate leases, where the real estate asset market with identical developers is modeled as a continuous time Nash equilibrium and the real estate leasing market is modeled as a contingent claim on the equilibrium building value.

Sequential investments are common in the real estate market, which introduces many options (e.g., wait, abandon, etc.) that can be analyzed with the real option methodology. Our paper relates to Rocha et al. (2007) where they develop a real options model for

investment analysis of residential building that determines the optimal investment strategy. They show when it is optimal to invest simultaneously or sequentially.

In this paper we analyze a shopping center investment, which differs from a residential building investment in several ways.

First, the main difference is the stochastic underlying process. In case of the shopping center it is the revenue from renting which is usually a percentage of sales that tenants generate. Therefore, the selection process of the store owners has a huge impact on the success of the entrepreneurship, while residential units can be sold to anyone with sufficient funds.

Second, local competition may have a long-term effect on the shopping center project than in the case of the residential building. For instance, a consumer that decides to buy a new residential unit is unlikely to change it to another offer from a competitor during the first years. However, the shop owners may move to another shopping center if a competitor's entrepreneurship has a higher traffic of consumers. This scenario can generate not only direct losses, such as losing a shop owner (i.e., losing immediately the revenue from the rent) but also indirect losses (i.e., reduction in the traffic of consumers; hence, reducing revenue from the rent of other stores, since rent is usually a percentage of sales). Therefore, the second stage of the original project might be subject to competitor's entry. Presence of competition in a particular region decreases the traffic in the original shopping center and revenues by a fraction (ω). We show that the entry of competition accelerates the second stage of the investment project.

Third, unlike investment in residential buildings, shopping center investment is an asset where the future payoff depends on the future stages of the investment. While investors in residential building have the main goal to sell all the apartments, investors in shopping centers are concerned not only with the process of renting or selling the stores, in order to launch the shopping center with the highest occupancy rate as possible, but also with the success of sales and traffic after inauguration, so that it can turn into a sustainable business. Hence, a residential building is an investment project with a finite life cash flow, while shopping center investment is a long-term business.

A shopping center investment is normally planned as a series of sequential decisions in order to diversify risk. The first phase reveals important managerial information related to future expansions or developments of the local potential market. However, the developers will only perceive the success/failure of the shopping center after finishing the construction and inaugurating the entrepreneurship, since they now can get a sense of the public's reaction and preferences. When the first stage is successful, the revenues increase due to the attractiveness of the potential market; hence, the following construction phases will be appreciated. In the opposite case, the developers can wait for a better moment before launching the next construction phase; this waiting option can help them reevaluate their prospects about the investment and update their decision with respect to potential competition. Another important aspect of planning the entrepreneurship as a multi-phase project is that it requires a lower initial capital outflow and previous phases may finance subsequent construction phases (i.e., the revenues of the previous phase can be used to finance the future construction phase).

In a feasibility study of a shopping center, the anchor tenants are extremely important, since they normally help to attract more customers to the mall. Hence, the decision of selecting good anchor tenants is vital for the success of the project. For instance, in Brazil, only 10% or less of the shopping centers are operating without anchor tenants. Since this reveals to be a good strategy to increase the traffic of the mall, the anchor tenants have usually special leasing contract (e.g., lower rents) or have the option to buy a space in the shopping center.

Many investment projects in shopping centers already consider the area for future construction phases, and analysts introduce usually the option concept intuitively in their evaluation of the project. For instance, strategies that take into account waiting, expansion or abandonment options are utilized commonly in every day life; in addition, these strategies are not guided by the discounted cash flows, but by subjective considerations or the expertise of the analyst. However, few analysts utilize the real option methodology in this market due to some disadvantages. For instance, Lander and Pinches (1998) states that the real option approach is not well known or understood by analysts, since they require mathematical skills to use them. Despite these disadvantages, the real option approach provides a better valua-

tion technique than the NPV methodology, since it provides a principled method that takes into account managerial flexibilities and uncertainties.

Our third contribution is to the literature on real estate, where we present a method that can be used by practitioners in the real estate industry. Empirical studies provide direct evidence that the value of the flexibilities is taken into account; however, the models that show how to take into account the value of managerial flexibilities in practice are limited. There are several options found in the shopping center market in Brazil, such as waiting option, information gathering option after the first launch, expansion and abandonment options. Furthermore, the competition among shopping centers plays an important role in our model. While the previous study does not take into account the effect of competition on the value of managerial flexibilities, we show that the fear of preemption by another firm can erode the option value. We combine theoretical models and industry experience to present a powerful tool that can simplify and improve the investment analysis in uncertain environments. Hence, it is important to establish a business culture in order to quantify these options objectively, identifying the uncertainties and most relevant options, and implementing strategies to manage them appropriately.

This work is organized as follows. Section 2 presents the uncertainties and managerial flexibilities in shopping center industry. Section 3 presents a methodology to calculate the best moment to launch the second stage of the project together with the value of the American option and the critical stopping curve that triggers investment. Section 4 presents a simulation of the model to verify the sensibility of some variables. Section 5 presents our concluding remarks.

2. Shopping Center Market and Real Options

The NPV Methodology has been one of the most widely used methodologies for investment analysis and valuation since the 70s. However, it proved to have major shortcomings. Dixit and Pindyck (1994) say that the assumptions of the NPV Methodology are grossly wrong because it considers the investment decision to be a now-or-never decision. In practice, de-

velopers have the flexibility to adjust, and even change, their future decisions when new information is provided and uncertainty decreases. This is called in the real options literature as the value of waiting. Schwartz and Trigeorgis (2004) show that managers overrule conventional NPV as they add an extra value such as operating flexibilities and strategic value of the project that it is not correctly calculated.

In this section we describe a managerial decision making process in the case of the shopping center project. We define different real options that developers have and we show how they can be implemented in the project valuation.

2.1. Feasibility Studies in Shopping Centers Projects

A shopping center investment is normally planned as a multiple phase project and as a series of sequential decisions to diversify risk. For simplicity, we focus here on a two stage project and make it as realistic as possible according to the industry practice. However, the model can be extended to multiple stage project. First stage of the project involves high sunk cost as normally at this stage the developer has to buy land and construct the first development. Furthermore, this stage is associated also with information gathering (e.g., market research) which also constitutes a significant cost of the project. The developer compares this cost against the future expected profits.

The first launch reveals important managerial information related to future expansion which affects the decision making process at the second stage. For example, the inaugurated first stage provides more accurate information (metrics) about public's reaction and preferences (i.e., the products they are consuming and the amount of people in this new shopping center). If the first stage is successful, the following construction phases will be appreciated and the revenues increase as well as the attractiveness of the potential market. However, the attractiveness of the area can attract potential competition. Therefore, the developer has to take into account also that the market might be shared. In the opposite case, the developers can wait for a better moment before continuing with the next construction phase and will, in the meantime, reevaluate their prospects about the investment.

Moreover, they can change their strategy about the mix of stores and in the extreme

scenario, even change the main focus of the shopping center. In this way, the first development provides a valuable option to obtain market information and generate waiting, expansion, switch or abandon options in the following phases of the project.

Another important aspect is that the launch of the shopping center investment in stages requires a lower initial capital outflow and may finance subsequent construction phases, since the revenues of the first stage will be used to build the next one. The presence of uncertainties and flexibilities is inherent to real option valuation, which allows the investor to analyze and manage strategic decisions optimally.

2.2. Embedded Real Options

Uncertainties are variables that we do not control (random variables), such as economic and technical variables. On the other hand, managerial flexibilities are variables that we can control (controlled variables) such as flexibilities to expand, diminish or abandon a project. The basic underlying principles of the real options analysis are flexibility (a right but not an obligation to do something that aggregates value; i.e., increases revenue or decreases losses), uncertainties, high sunk cost, and irreversibility or partial irreversibility.

The presence of uncertainties and flexibilities which is inherent to real option valuation, allows the investor to analyze and manage strategic decisions optimally. The relevant options in a shopping center investment are:

- Information option: How the success/failure of the first stage influences the results of the next stages;
- Deferral (Waiting) option: Postpone the start of the next stage if the market is not favorable;
- Expansion option: Proceed to the next stage when the market is favorable for a new development;
- Switch option: Change the mix of the stores in response to the market's reaction (e.g., create a shopping center specialized in furniture);

- Density option: Decide the size (area) to be used in the shopping center, subject to restrictions under local legislation;
- Abandon option: Decide not to proceed to the next stage due to excessive construction and contractual costs.

3. Methodology

In order to evaluate the value added by the real option model herein (Appendix A.), we evaluate a strategy that considers managerial flexibilities and competition against another strategy based on a static decision making process. The former strategy encompasses a two stage construction of a shopping center where managers launch the first stage and have the flexibility to wait for the best moment (i.e., when the market is favorable and due to the fear of competition) to launch the second stage; if the market is not favorable for launching the second stage, the managers have also the option to abandon the expansion project. The latter strategy includes the same two stage construction of a shopping center where the second stage is always launched after a fixed period of time (e.g. 10 years after the first stage is launched).

3.1. Assumptions and Basic information

A shopping center project starts normally with a negotiation process between investors and the land owner in order to decide the price of the land. A feasibility study of the shopping center project is typically used to determine the best price in this negotiation process. In addition, the feasibility study relies also on market research and a first draft of the architecture project (the architect leaves usually some space for the expansion of the shopping center). Hence, the following parameters are estimated by the feasibility study:

- Rent area for small tenants: The constructed area for small tenants, corresponding to the square meters of floor space that the developer is renting for small tenants;
- Rent area for anchor tenants: The constructed area for anchors, corresponding to the square meters of floor space that the developer is renting for anchor tenants;

- Calculated Rent Area: Total weighted area that takes into account different weights for the area of the anchor stores and the other stores. This parameter is extremely useful to analyze the performance of the shopping center;
- Equivalent area: Constructed area used to calculate the construction cost. For example, the construction cost of the mall is more expensive than the construction cost of the parking lot; therefore, the equivalent area is a weighted area that uses different weights for different parts of the shopping center project;
- Number of kiosks: The project has usually various kiosks placed in strategic points in the mall; this is an important alternative to generate more revenue for the investors;
- Construction Cost: A present discounted value of the total cost related to the construction (i.e., the equivalent area times the construction cost per squared meter taking into account the time to build the shopping center). In addition, this cost includes usually an administration fee for the Construction Company as well as expenses related to the architecture project.
- Land cost: There are several ways to buy the land, such as cash, loan, and exchange contracts (on the revenue of the shopping or for other realty). The taxable price of the land, also known as the base value, has some associated costs such as taxes, commissions, demolition and etc. In addition, the final price of the land is usually the last thing that investors agree with the owners in the negotiation process; this is due to the fact that other expenses and predicted revenue are important factors to determine the maximum value that they can pay for the land.
- Assignment of right to use: The rent is not the only revenue in the cash flow; in addition to the rent, the small tenants have also to pay to use the space in the shopping. This extra revenue is usually calculated by multiplying the area of the store by a price per squared meter, according to the rent speed and the amortization table. The rent speed is a variable that determines the percentage of stores that are leased for each period of time.

- Minimum Rent Revenue: The present value of the revenue flows as the minimum rent that tenants have to pay every month. A lease is normally based on a percentage of total sales of a store. However, the tenants are usually obliged to pay a minimum amount when the sales are below a threshold.
- Percentage rent: The present value of the revenue flow as the difference between the total rent and the minimum rent. The developers use their experience to estimate this parameter.
- Parking lot revenue: The present value of the revenue flow that developers receive from the parking lot. In Brazil, the parking lot revenue in the majority of shopping centers represents almost 10% of the total monthly revenue.
- Operational revenue: The present value of the total revenue when the shopping is operating; hence, this value is the sum of the minimum rent, percentage rent and parking lot revenue.
- Operational cost: The present value of all costs when the shopping is operating (e.g., administrative costs, energy, water and others).
- Net operational revenue: The present value of the operational revenue minus operational costs.

In practice, shopping centers are normally a multi-stage project where the investment of each stage starts in different periods of time. Hence, multi-stage projects may take a long time to end and construction costs are not paid uniformly during this period. This work takes into account the fact that the construction cost curve is not uniformly distributed over time, while previous works have made unrealistic assumptions of this cost (such as (Titman 1985) and (Williams 1991)).

3.2. First Stage

The first stage relies on information about the market, such as revenue per-capita, potential market, target consumer, potential anchors, and potential mix of stores. Market research is typically used to provide information about the potential market. In addition, an architecture project and consequently its costs are also important in order to analyze the feasibility study of the investment.

We assume also that the leasing contracts of the stores are usually signed before the shopping center starts to operate; however, the contracts are signed during different periods of time. This assumption is modeled via a rent speed, a typical variable that defines how fast the project's stores are leased over time. The rent per square meter is one of the main uncertainties of the shopping center market, and our model approximates this variable by a triangular probability distribution as follows:

$$y^w \sim \text{Triang}[y_{min}; y; y_{max}] \quad (1)$$

where y_{min} and y_{max} are the boundary conditions and can be defined based on the market research.

The methodology to analyze the first stage of the project is based on four steps: First, the rent/m² of the small tenants is randomly selected with a triangular distribution (Equation 1). Second, a discounted cash flow is performed. Third, the success or failure of the first stage is computed based on some indicators of success (Table 1 presents an example of the indicators of success). Fourth, if the first stage is considered a success, we proceed to the second stage immediately. Otherwise, the construction of the second stage is deferred for a given period of time.

3.3. Real Options and Second Stage

This section describes the scenario where managerial flexibilities and competition are taken into account in the decision making process. A developer, under the fear of preemption, launches only the second stage when the market is considered to be favorable. This is

Table 1: **Indicators of Success.** The success or failure at the first stage is calculated on the basis of parameters that establish the success. These parameters are calculated based on the industry experience.

Indicators of Success	
Net Revenue / Operational Revenue (3rd year)	$\geq 80\%$
IRR annually	$\geq 15\%$
NPV / PV Revenue	$\geq 20\%$

equivalent to an American call option with preemption fear (Appendix A.) that can be exercised at any time by paying the construction cost of the second stage.

Following the usual hypothesis in option pricing, we assume that the sales of the shopping center (i.e., how much the tenants sell) follows the geometric Brownian process in Equation 2, where dz is the Wiener process, r is the risk free rate, δ is the dividend yield and σ is the volatility, all parameters are annualized.

$$\frac{dS}{S} = (r - \delta)dt + \sigma dz \quad (2)$$

The volatility σ can be estimated by OLS from the returns of the historical data. The parameter δ of the option is equivalent to the cash flow that the developers do not receive for not exercising the expansion option immediately. This cash flow can be, for example, the consumers that the shopping center loses for not expanding the second stage immediately. In addition, δ can also represent an opportunity cost of delaying the second stage of the project. As this measure is not precisely observed, we have to typically perform a sensibility analysis in order to select a value for δ .

In order to calculate the critical net operational revenue from where the investment in the second stage becomes optimal (also known as optimal stopping threshold), we utilize the analytic approximation for the American call option, presented in Appendix A. This is an extension of the Barone-Adesi and Whaley (1987) approximation, where the fear of

preemption is added. Equation 3 is utilized to obtain the optimal stopping threshold S^* , in which the parameters are explained in Appendix A.

$$S^* - X_2 = call(S^*, T) + [1 - e^{(b-r)(T-t)}N(d_1(S^*))]S^*/\gamma \quad (3)$$

The leasing contract stipulates that the rent is a percentage of the sales. Due to this proportionality, the rent/m² also follows a Geometric Brownian Motion with the same volatility as the sale price/m². In other words, due to the proportionality between sales and rent/m², we can prove by Itô's Lemma that the rent/m² follows the same stochastic process in Equation 2.

Let $S_1(y^w, \theta)$ and $S_2(y^w, \theta)$ be the present value of the net operational revenues of the first and second stage respectively, where y^w is the rent/m² and θ is the characteristics of the shopping development, such as construction time and number of stores. Let $S_2^E(y^w, \theta, \omega, \lambda)$ be the expected value of $S_2(y^w, \theta)$, as shown in Equation 4, where λ is the rate at which a competing firm can start the construction of a new shopping center in the same region, and ω represents the change in revenues when firms have to share the same region and compete. The option to proceed to the second stage (i.e., expand the shopping center) is exercised when $S_2^E \geq S^*$.

$$S_2^E(y^w, \theta, \omega, \lambda) = S_2 * (1 - \lambda) + S_2 * (1 - \omega) * \lambda \quad (4)$$

Let X_1 be the present value of construction cost minus the assignment of right to use of the first stage, and X_2 be the present value of the difference between construction costs and the assignment of right to use of the second stage. Equations 5 and 6 are the net present value of the static decision making strategy and the real option strategy respectively. Hence, the difference between the two strategies is the value added by the option.

$$NPV_1 = \mathbb{E}[S_1(y^w, \theta) - X_1] + \mathbb{E}[S_2^E(y^w, \theta, \omega, \lambda) - X_2] \quad (5)$$

$$NPV_2 = \mathbb{E}[S_1(y^w, \theta) - X_1] + \frac{1}{N} \sum_{w=1}^N F[S_2^E(y^w, \theta, \omega, \lambda) - X_2, t] \quad (6)$$

Note that the last term of Equation 6 is the investment opportunity of the second stage, in which the waiting and abandon options are incorporated. In addition, Equation 6 is obtained numerically with an algorithm that runs N iterations of the following steps: (i) as described in the previous section, the rent/m² is randomly selected with a triangular distribution (Equation 1); (ii) a discounted cash flow is performed in order to obtain $S_1(y^w, \theta) - X_1$; (iii) the success or failure of the first stage is computed based on the indicators of success; (iv) if the first stage is considered a success, we proceed to the second stage immediately; (v) otherwise, the construction of the second stage is deferred and the rent/m² follows a Geometric Brownian Motion; (vi) the option is only exercised when $S_2^E \geq S^*$ (within a finite time); (vii) otherwise, $F = 0$.

4. Simulations

In order to empirically test the model presented in the previous section, we developed a VBA code (available upon request) to simulate several strategies incorporating real options where the fear of preemption changes.

In practice, managers utilize the NPV to value investment projects and assume usually that a second stage is always launched after a fixed period of time. In contrast to the standard NPV valuation, the real option approach assumes that after the first stage is launched, the firm can gather information and revise assumptions about the future expansion or contraction. In other words, while time passes, more information is revealed and the firm can make better decisions about future actions. The firm has therefore a waiting option in order to launch the second stage (finite time American option). Hence, the option to defer can add value to the project because of the uncertainty about the future.

Table 2 presents the parameters of the base case scenario that were used in the implementation of the model. Recall that the developers define these parameters based on a market research and a project drawn by an architect.

Table 3 illustrates the results of the investment analysis with the real option strategy (based on the parameters in Table 2). We ran 1 000 simulations to approximate the results.

Table 2: **Parameters of the Model.** The parameters of the model are based on a shopping center in Brazil.

Rent Areas:		
	Anchor tenant	10 000 m ²
	Megastores	3 000 m ²
	Small tenants	15 000 m ²
Rent/m²:		
	For small tenants	y^w (triangular distribution)
	y_{min}	BRL 30/ m ²
	y_{max}	BRL 60/ m ²
	For megastores	y^w x 40%
	For anchor tenants	y^w x 20%
Assignment Right of Use:		
	Right of use	BRL 1200 / m ²
Kiosk:		
	Number of units	30
	Monthly rent per unit	40 x y^w
Rent speed:		
	Launch	25%
	During the first year	25%
	During the second year	45%
	Vacancy	5%
Investments cost at the first stage:		
	Construction	BRL 50 000
	Others	BRL 5 080
Investments cost at the second stage:		
	% Construction	BRL 26 250
Land:		
	% of Net Revenue	30%
Operational Expenses:		
	% of Revenue	24%
	Volatility (σ)	5.43%
	Dividend yield (δ)	6%

We present the different scenarios where the probability of a competitor arriving (λ) spans from 0 to 80%. We report also the optimal time to start the second stage (in months). In the first case, where $\lambda = 0\%$ the value of the option is BRL 2 787 912 and the second stage should be launched after 150 months. When the probability of the competitor arrival increases then the value of the option decreases and the timing of the launch is accelerated. This result is

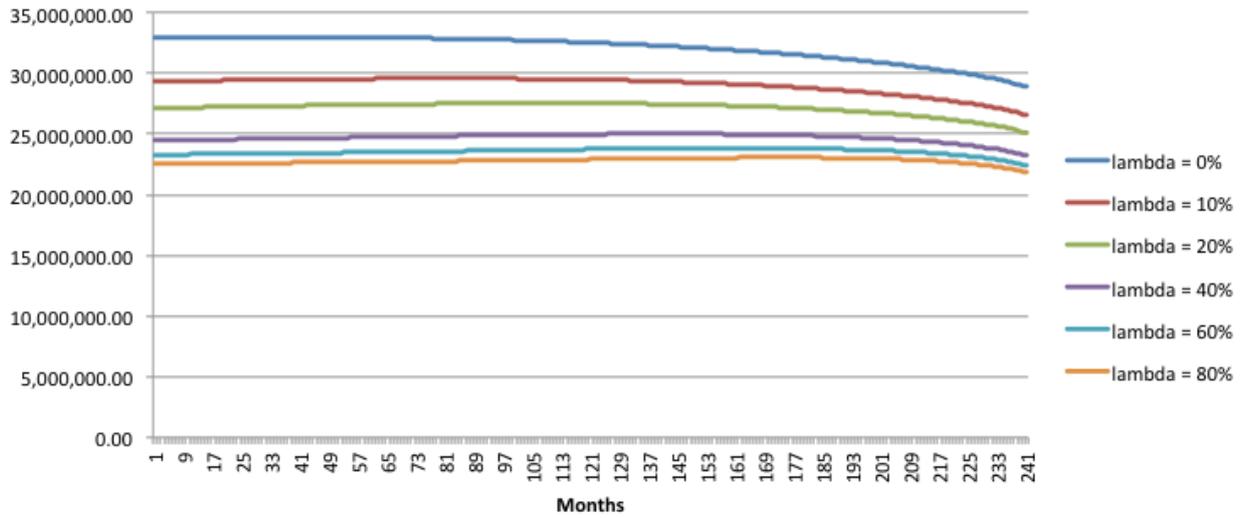
Table 3: **Simulation Results.** We assume that the cash flow discount rate is 15% and the discount rate of NPV_2 is R . We ran the simulation 1000 times. λ is the probability of a competitor arriving.

	λ (in %)					
	0	10	20	40	60	80
Average of the value added by the option (BRL)	2,787,912	2,515,985	2,354,387	2,242,574	2,062,094	1,693,771
Average value of the project (BRL)	9,373,344	8,920,845	8,862,527	8,598,692	7,979,477	7,464,622
Standard Deviation (BRL)	7,691,987	7,707,154	7,621,901	7,598,431	7,844,739	7,760,278
N ^o of times that the expansion option is exercised	812	852	891	927	912	891
% of times that the expansion option is exercised	81%	85%	89%	93%	91%	89%
Average N ^o of months to start 2 nd stage	150	130	118	102	98	97

consistent with the real options theory that competition erodes the value of the option and speeds out investment timing.

Figure 1 illustrates the optimal stopping threshold S^* as a function of time. We plot the threshold for different values of λ . The relationship between the investment threshold and time is negative. Shorter remaining life of the option accelerates the investment possibility. The explanation of that effect is that when the life of the option is finite the effect of the uncertainty on the option value can be non-monotonic (see for example Gryglewicz, Huisman, and Kort (2008)). Therefore, the optimal threshold drops with time. The investment threshold decreases when the probability of competitor's arrival increases. This result is consistent with the studies highlighting the role of competition.

Figure 1: **The Optimal Stopping Threshold, S^*** (in BRL), for different values of λ



5. Conclusion

Shopping center investments in emerging economies are characterized by low liquidity, slow payback and high sunk costs. Moreover, there are several uncertainties related to demand, sales price, rent prices, land costs, and regulatory and local government risks (authorization, occupancy permits, etc.), which increases the risk of the investment project. In many cases, entrepreneurs have to start the second stage of the project due to competition. Therefore, it is very important to create a methodology that calculates the optimal moment to launch the second stage of the project which takes into account the potential threat of competitor's entry. The model presented in this work uses the American Option to evaluate Shopping Center investments. The NPV methodology is used by many analysts to evaluate Shopping Center projects. The Discounted Cash Flow (DCF) (or the naive NPV methodology) considers the decisions as static and does not take into account managerial flexibilities and uncertainties.

The methodology in this paper determines the value added by the option to the project, together with the critical net revenue (sales from tenants) that triggers investment (optimal threshold curve), and the best moment to launch the second stage.

In practice, many shopping center analysts intuitively use the concept of options in their

analysis. However, they do not use a systematic and consistent project analysis that incorporates the real option theory. In Brazil during the 70's, the profit margin of a typical project was around 50% and now is down to roughly 20%. Therefore, it is important to establish a managerial culture in order to quantify the value of these options objectively and provide effective risk management. The proposed model proved to be a better valuation technique than the discounted cash flow approach.

However, real option methodologies have not been generally applied due to some disadvantages. Among them, the main disadvantage, also stated by Lander and Pinches (1998), is that corporate managers do not fully understand the real option models, since they require specific mathematical skills. In addition, the complexity of the model increases when a project has more sources of uncertainty and embedded options.

Despite the disadvantages, real option is a methodology that provides more valuable information to decision-making processes than the NPV methodology. In this paper, we create a useful tool to handle several sources of uncertainty and managerial flexibilities without significantly increasing the complexity of the model. This methodology combines specific knowledge in shopping center industry, real option approach and simulation technique to evaluate shopping center investments.

An interesting extension of the current version of the model would be to calculate the investment timing during different economic states. In particular, firms' cash flows can depend on small idiosyncratic shocks (such as GBM) and on large aggregate economic shocks (two-state Markov chain). During economic boom the investment threshold is lower than during recession ($\bar{S}_B < \bar{S}_R$). The merger can be triggered in two ways. First, either during recession or boom the stochastic process can hit the threshold \bar{S}_B if economy is in an expansion or \bar{X}_R if the economy is in a contraction. Second, the change in the business cycle can trigger the investment. Therefore, we can expect more activity during economic booms than recessions with major developments after positive economic shocks.

Appendix A. Value of the finite-time American option with preemption fear

This is an extension of the Barone-Adesi and Whaley (1987) approximation, and the assumptions are consistent with option-pricing model of Merton (1973) and Black and Scholes (1973). In particular, we assume that the risk-free rate r and that the cost-of-carry b are constant. The cost-of-carry for non-dividend paying stock is assumed to be equal to the risk-free rate ($b = r$). For stocks paying dividends, the cost-of-carry is equal to the risk-free rate less the dividend yield ($b = r - d$).

In the real options literature it is often assumed that the state variable follows a geometric Brownian motion process as follows:

$$dS = \mu S dt + \sigma S dz \tag{A.7.}$$

where μ is a deterministic drift, σ is volatility, and dz is the Wiener process.

The crucial difference to the Barone-Adesi and Whaley (1987) approximation of an American option value is the introduction of an additional parameter $\lambda > 0$, where λ is the hazard rate related to the Poisson process (the probability of a fixed size jump occurring to the claim value).

In other words, we assume that a competing firm can start the construction of a new shopping center in the region at a rate λ . When $\lambda = 0$, the probability of a competitor starting the construction in the same region over a unit time interval is zero. The resulting partial differential equation (PDE) is the same as Barone-Adesi and Whaley (1987). When λ increases then the probability of a competitor entering the market increases.

We assume now that exercising American option to invest is conditional on whether the competitor starts the construction of a shopping center or not. When the competitor arrives then both firms have to share the same market. Therefore, the revenues and value of the project of the incumbent shopping center drops to $(1 - \omega)V(S)$, where $\omega \leq 1$ and represents the change in revenues when firms have to share the same region and compete.

Before exercising the option to invest, the value of the project should satisfy the following equation:

$$V(S) = \max_{\mathcal{T}_S} \mathbb{E}[\mathbb{1}_{\mathcal{T}_S < \mathcal{T}_C} e^{-r\mathcal{T}_S} (V(S^*) - X_2)] \quad (\text{A.8.})$$

where \mathcal{T}_S is the time of option exercise and \mathcal{T}_C is the time when the competitor arrives, and $\mathbb{1}$ is an indicator function taking values one when the event occurred and zero otherwise.

Partial differential equation including both the diffusion parts and the jump to the project (eq.4 from Barone-Adesi and Whaley) for $S < S^*$ is now:

$$rV = 1/2\sigma^2 S^2 V_{SS} + bSV_S + V_t + \lambda[(1 - \omega)V - V] \quad (\text{A.9.})$$

Re-witting:

$$(r + \lambda\omega)V = 1/2\sigma^2 S^2 V_{SS} + bSV_S + V_t \quad (\text{A.10.})$$

Or where $R = r + \lambda\omega$:

$$RV = 1/2\sigma^2 S^2 V_{SS} + bSV_S + V_t \quad (\text{A.11.})$$

Then, applying the above partial differential equation and following Barone-Adesi and Whaley (1987) approximation, the value of the European call option changes. When the incumbent shopping center has preemption fears the risk-free rate r changes to R . The value of a European call option is now:

$$call(S_0, t) = S e^{(b-R)(T-t)} N(d_1) - X e^{-R(T-t)} N(d_2) \quad (\text{A.12.})$$

$$d_1 = \frac{\ln(S/X) + (b + 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}} \quad (\text{A.13.})$$

$$d_2 = d_1 - \sigma\sqrt{T - t} \quad (\text{A.14.})$$

The American call option premium is given by the following equation:

$$Call(S, T) = \begin{cases} call(S, T) + A(S_0/S^*)^\gamma & S < S^* \\ S - X & S \geq S^* \end{cases} \quad (\text{A.15.})$$

where $A = (S^*/\gamma)[1 - e^{(b-R)(T-t)}N(d_1(S^*))]$, $\gamma = [-(\beta - 1) + \sqrt{(\beta - 1)^2 + 4v/h}]/2$, $v = 2R/\sigma^2$, $\beta = 2b/\sigma^2$, and $h = 1 - e^{-R(T-t)}$ and $s^*(t)$ is the solution to:

$$S^* - X_2 = call(S^*, T) + [1 - e^{(b-r)(T-t)}N(d_1(S^*))]S^*/\gamma \quad (\text{A.16.})$$

References

- Barone-Adesi, Giovanni, and Robert E. Whaley, 1987, Efficient analytic approximation of american option values, *The Journal of Finance* 42, pp. 301–320.
- Black, Fischer, and Myron S Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–54.
- Brennan, Michael J, and Eduardo S Schwartz, 1985, Evaluating natural resource investments, *The Journal of Business* 58, 135–57.
- Bulan, Laarni, Christopher Mayer, and C. Tsurriel Somerville, 2009, Irreversible investment, real options, and competition: Evidence from real estate development, *Journal of Urban Economics* 65, 237 – 251.
- Capozza, Dennis, and Sick Gordon, 1994, The risk structure of land markets, *Journal of Urban Economics* 35, 297–319.
- Capozza, Dennis, and Yuming Li, 1994, The intensity and timing of investment: The case of land, *American Economic Review* 84, 889–904.
- Cunningham, Christopher R., 2006, House price uncertainty, timing of development, and vacant land prices: Evidence for real options in seattle, *Journal of Urban Economics* 59, 1 – 31.
- Dixit, Avinash K., and Robert S. Pindyck, 1994, *Investment Under Uncertainty* (Princeton University Press).
- Grenadier, Steven, 1995, Flexibility and tenant mix in real estate projects, *Journal of Urban Economics* 38, 357–378.
- , 1996, The strategic exercise of options: development cascades and overbuilding in real estate markets, *The Journal of Finance* 51, 1653–1679.

- , 2005, An equilibrium analysis of real estate leases, *The Journal of Business* 78, 1173–1214.
- Grovenstein, Robert, James Kau, and Henry Munneke, 2011, Development value: A real options approach using empirical data, *The Journal of Real Estate Finance and Economics* 43, 321–335.
- Gryglewicz, Sebastian, Kuno J.M. Huisman, and Peter M. Kort, 2008, Finite project life and uncertainty effects on investment, *Journal of Economic Dynamics and Control* 32, 2191 – 2213.
- Lander, Diane M., and George E. Pinches, 1998, Challenges to the practical implementation of modeling and valuing real options, *The Quarterly Review of Economics and Finance* 38 (Special Issue), 537–567.
- McDonald, Robert, and Daniel Siegel, 1986, The value of waiting to invest, *The Quarterly Journal of Economics* 101, 707–27.
- Merton, Robert C., 1973, Theory of rational option pricing, *Bell Journal of Economics* 4, 141–183.
- Myers, Stewart C., 1977, Determinants of corporate borrowing, *Journal of Financial Economics* 5, 147–175.
- Quigg, Laura, 1993, Empirical testing of real option-pricing models, *Journal of Finance* 48, 621–40.
- Rocha, Katia, Luciana Salles, Francisco Augusto Alcaraz Garcia, Jose A. Sardinha, and Jose P. Teixeira, 2007, Real estate and real options - a case study, *Emerging Markets Review* 8, 67 – 79.
- Schwartz, Eduardo S., and Lenos Trigeorgis, 2004, *Real Options and Investment under Uncertainty. Classical Readings and Recent Contributions* (Cambridge: The MIT Press).

Titman, Sheridan, 1985, Urban land prices under uncertainty, *American Economic Review* 75, 505–14.

Trigeorgis, L., 1996, *Real Options: Managerial Flexibility and Strategy in Resource Allocation* (Cambridge: The MIT Press).

Williams, Joseph T, 1991, Real estate development as an option, *The Journal of Real Estate Finance and Economics* 4, 191–208.

Yung Medeiros, Priscilla, 2003, Applications of real options in the real estate market focusing the city of rio de janeiro, *Brazilian Review of Finance* 1, 45–87.