Emergence of Social Balance in Signed Networks

Andreia Sofia Teixeira, Francisco C. Santos and Alexandre P. Francisco

Abstract Social media often reveals a complex interplay between positive and negative ties. Yet, the origin of such complex patterns of interaction remains largely elusive. In this paper we study how third parties may sway our perception of others. Our model relies on the analysis of all triadic relations taking into account the influence and relations with common friends, through large-scale simulations. We show that a simple peer-influence mechanism, based on balance theory of social sciences, is able to promptly increase the degree of balance of a signed network—with balance defined as the fraction of positive cycles—irrespectively of the network we start from. Additionally, our results indicate that the tendency towards a balanced state also depends on the network connectivity and on the initial distribution of signs.

Keywords Balance theory · Network analysis · Social networks

1 Introduction

Signed networks are networks where the links have a sign expressing some positive or negative tie between individuals [1–8]. It is well-known that in social networks one can be friendly or unfriendly with others and that this can change over time. Moreover, individuals also shape and reshape their social environment themselves and are responsible for the specific features that characterize their social network [9–12]. Social balance theory, a concept developed by Heider [13], and later adapted to a graph-theoretic model by Cartwright and Harary [1], states that in a triad, the relations of friend-enemy tend to converge to two balanced states: “the friend of my

A.S. Teixeira (✉) · F.C. Santos · A.P. Francisco
INESC-ID/Instituto Superior Técnico, Universidade de Lisboa, Lisbon, Portugal
e-mail: sofia.teixeira@tecnico.ulisboa.pt

F.C. Santos
e-mail: franciscocsantos@tecnico.ulisboa.pt

A.P. Francisco
e-mail: aplf@tecnico.ulisboa.pt

B. Gonçalves et al. (eds.), *Complex Networks VIII*,
Springer Proceedings in Complexity, DOI 10.1007/978-3-319-54241-6_16

friend is my friend" and "the enemy of my enemy is my friend", otherwise there will be tension between them.

In 1946, Fritz Heider published an initial study about how affective ties—as to like, to love, to esteem, etc., and their opposites—would influence interpersonal relations [13]. These simple cognitive configurations between people and objects led to the conclusion that a triad is balanced if the three links are positive, or if two are negative and one positive, otherwise tension would emerge. This was a primary approach to social balance. Later, Cartwright and Harary, extended this notion of balance to a graph—structural social balance—and used the concept of signed graphs, where the ties between the individuals have a positive or a negative sign, to express those kind of relations [1, 2]. They extended the concept of triad to a cycle, allowing cycles with more than three edges, and defining the sign of the cycle as the product of the signs of its edges. A cycle is then considered balanced if the product is positive. They also introduced the concept of degree of balance of a signed network as the ratio of the number of positive cycles to the total number of cycles. Let G be a signed graph, $c(G)$ be the number of cycles of G, $c_+(G)$ be the number of positive cycles of G, and $b(G)$ be the degree of balance of G. Then:

$$b(G) = \frac{c_+(G)}{c(G)}$$

In our work we use this measure applied to triads, that is, cycles of size 3.

Following the work of Cartwright and Harary, in 1967, Davis [3] studied the relation between clustering and structural balance in graphs. The main question was about what conditions were necessary and sufficient for the graph to be separated into two or more subsets of nodes, where each positive edge would link two nodes of the same subset and a negative edge would link nodes from different subsets. Those conditions were: a signed network is clusterable if and only if the network does not contain any cycle with exactly one negative link. This introduced the notion of *weak balance theory* as it allows for cycles/triads to have all signs negative, meaning that "the enemy of my enemy can be an enemy", allowing more than two subsets to be created. The main conclusion was that all balanced graphs are clusterable.

Global structural balance has also been studied. Doreian et al. [4] created an agent-based simulation model based on two levels: a micro-level that explores Heider's theory at an individual level, to minimize individual tension; a macro-level that explores Cartwright and Harary's at a group level dynamics. This simulation model is only for small groups dynamics as the designed variables have complicated impacts. Facchetti et al. [7] implemented an algorithm for ground-state calculation in large-scale Ising spin glasses, to compute the global level of balance in large undirected networks. And recently Estrada and Benzi [8] published a study about structural social balance in directed networks.

In this work, we evaluate how the relations between individuals change over time, based on the relations with common friends, and if those changes converge to a balanced social structure. We present a simulation model that, at each iteration, evaluates if the sign between two individuals must change to minimize tension across

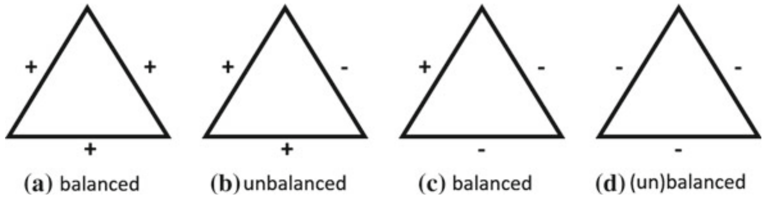


Fig. 1 Social Balance Theory, by Cartwright and Harary [1]. The triads are considered balanced if the product of the signs are positive. Davis introduced the weak balance structure that considers all triads but the second to be balanced

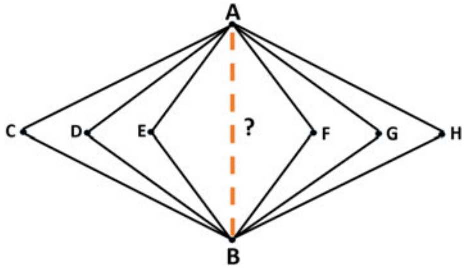
triads. A triad is considered balanced if its edges have the signs $\{+, +, +\}$ or $\{+, -, -\}$, meaning that the product of its signs has to be positive—see Fig. 1. We consider that the polarity of the relations is reciprocal, considering only undirected networks. We run our simulations with the original distribution of signs of each chosen dataset, but also with a random distribution of the signs, both in the same proportion as in the original network and in equality proportion of positive and negative links. We observe that the evolution of the signs between individuals involved in triads converge towards an increase of structural balance, minimizing the tension between the individuals, but also that the final dominant triads depend on the initial distribution of the signs.

2 Methods

Let $G = (V, E)$ be an undirected and weighted graph (signed social network), with $n = |V|$ vertices (individuals) and $m = |E|$ edges (ties), and with edges weight between two individuals (a,b): $w(a, b) = w(b, a) = 1$, if it is a positive tie, $w(a, b) = w(b, a) = -1$ if it is a negative tie. For each pair of individuals with friends in common, our model will count how many of those relations contribute with a positive or negative sign, based on balance theory.

Looking into the example illustrated in Fig. 2: given a network let us consider individuals A and B that have C, D, E, F, G, H as friends in common. We now evaluate if the product between $w(A, C)$ and $w(C, B)$ is positive or negative, and the same for the other neighbours. Because we want to reduce tension in triads, the sign between individuals A and B will depend on a majority count between positive and

Fig. 2 What will be the sign between A and B? It will depend on the majority of the signs of the products of each vertex A and B with each neighbour



negative products of the other relations in the triads related to that link. Given the sign between individuals A and B, $w(A, B)$, it will only be updated if the majority of the counts of the products have an opposite sign of the present sign. Illustrating a little bit more: if $w(A, C) = -1$ and $w(C, B) = -1$, the product is equal to 1, so if we want the triad to be balanced we count this as a positive contribution, i.e., if the sign only depended on this triad it would be positive. If $w(A, C) = -1$ and $w(C, B) = 1$, the product is equal to -1, so if we want the triad to be balanced $w(A, B)$ would have a negative contribution in the count. If the sign only depended on this triad $w(A, B)$ would be negative. We remind that a triad is balanced if the product of the signs of its edges is positive. We do this count for each neighbour in common. In other words: $w(A, B)$ will be the sign corresponding to the majority of positive or negative contribution counts.

The algorithm runs in two parts, as follows:

```
for each user u do
    for each neighbour n do
        collect the friends in common

        for each friend in common c do
            if the product between w(u, c) and w(n, c) == 1 then
                pos(u, n) ⟵ pos(u, n) + 1
            else
                neg(u, n) ⟵ neg(u, n) + 1
            end if
        end for
    end for
end for
for each edge (a,b) do
    if pos(a,b) == neg(a,b) then
        there is no update and w(a, b) stays the same
    end if
    if pos(a,b) > neg(a,b) then
        w(a, b) ⟵ 1
    else
        w(a, b) ⟵ −1
    end if
end for
```

In the end of each iteration—an iteration corresponds to the execution of both parts—we count the proportion of each four possible triads and calculate the degree of balance of the network. The simulations run until there are no more changes in the edge signs or until it reaches a given threshold on the counts changes. Note that changes are independent, i.e., they are synchronous and do not depend on other possible updates. We stop the simulation when either the average of the fraction of the edges signs changed in the last two iterations is below 10^{-2}, or its difference for the last three iterations is below 10^{-4}. These thresholds were determined experimentally.

Table 1 Networks used in the simulations

Network	# Nodes	# Edges	% Edges +	% Edges −	# Triangles
HighlandTribes	16	58	50.00	50.00	68
Epinions	131 828	708 507	83.25	16.74	4770102
Slashdot	82 144	498 532	76.41	23.59	571127

3 Results and Discussion

In these experiments we used well-known signed social networks: Highland Tribes, the signed social network of tribes of the GahukuGama alliance structure of the Eastern Central Highlands of New Guinea, from Kenneth Read (1954). The network contains sixteen tribes connected by friendship and enmity[1]; Epinions, a who-trust-whom online social network of a general consumer review site Epinions.com[2]; and Slashdot, a website which allows users to tag each other as friends or foes.[3] We also created cliques with different sizes just to compare complete connected networks with Epinions and Slashdot that are large-scale sparse networks. Because Epinions and Slashdot datasets are directed networks, we performed some operations in these networks to make them undirected. We analysed each network and if some relation had a conflict—one edge in one direction positive, and in the other direction negative—we removed that edge, keeping only the relations that are reciprocal.

In Table 1 we can find the characteristics of each network. We processed each social network in three different ways: (1), we started by running the simulations with the networks as they were after removing conflicting edges; (2), for each network we randomly distributed the signs of the edges in the same proportion as in the original network; (3) for each network we distributed randomly and evenly positive and negative signs, i.e., 50% of positive edges and 50% of negative.

In Fig. 3 we present the results of the simulations. It contains the initial and final distribution of the four possible triads and of the degree balance. As we can observe, the initial distribution of triads in the *Random* and in the *Evenly* networks are very different when compared to the original. Even when maintaining the initial proportion of positive and negative links, this means that there are some triads that are overrepresented in the original network, which indicates that the way signs are distributed initially has direct impact in the structural balance.

We can observe that having a dominant quantity of positive or negative links makes a network to converge to a dominant all-positive triads: (1) if individuals like each other and the friends in common also like each other, then there is no social reason to change; (2) if individuals do not like each other or the friends in common, then there are triads in tension and the update rule forces signs of all-negative triangles

[1] http://konect.uni-koblenz.de/networks/ucidata-gama.

[2] https://snap.stanford.edu/data/soc-sign-epinions.html.

[3] https://snap.stanford.edu/data/soc-sign-Slashdot090221.html.

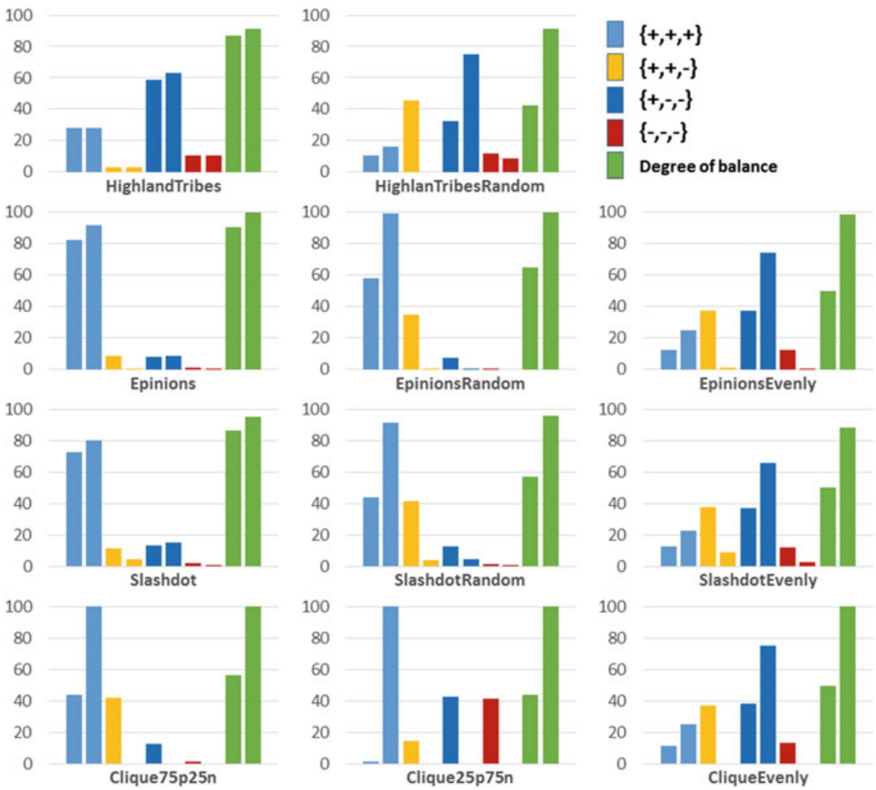


Fig. 3 Simulations results. Each pair of columns corresponds to the initial and final distribution of each triad, except for the last pair which represents the initial and final degree of balanced of the network. As seen in Fig. 1, only the first and third triads are considered balanced. *Random* means that the signs were distributed randomly in the same proportion of the original network and *Evenly* means that the signs were distributed randomly with 50%–50% of positive-negative signs. HighlandTribes does not have Evenly because the distribution is already 50%–50%. We omitted the size of the clique, but we used sizes between 8 and 64 and the results were the same

to change to positive. In all networks there is an increase of balanced triads, but depending on the initial distribution of signs, the dominant triads are different. All networks with 50%–50% of positive and negative signs, instead of converging to the same initial dominant triads, converge to the two-negative one-positive triads. There are two reasons for this to happen: we have a high initial distribution of the triad $\{+, +, -\}$ that will always change to $\{+, -, -\}$; the decrease of the distribution of the triad $\{+, +, +\}$, when comparing to the original networks, also indicates that there is not enough all-positive triads to compete with the new dominant $\{+, -, -\}$. This leads to the conclusion that initial distribution of positive and negative ties has direct impact in how signs can evolve and, again, in the degree of balance.

Making the signs evolve based on the balance theory criteria will always force the individuals of the network to act towards a minimization of tension. There is a strong tendency towards balance, but not always enough to achieve 100% balance. This happens in the non-fully connected networks, usually when the changes reach the threshold on the number of changes. We observe that the achievement of total degree of balance may depend on the connectivity of the network—fully connected networks eventually converge as can be seen in cliques. This last conclusion was already derived theoreticly in previous works by Antal [14] and Arnout van de Rijt [15]. With different approaches, both come to conclusion that in a complete connected networks, when updating triads with the goal of minimizing imbalance, a balanced state is achieved.

4 Conclusions

Network Science [13, 16] has provided key insights on how individual states, from individuals choices [17–19], epidemic states [20], strategic behaviours [9, 21–24], and opinions [25], among other traits, are locally influenced by their social ties and by the overall topology of interactions within a population. While the dynamics at the level of nodes is crucial, analogous dynamics occurs at the level of states and weights of links [5, 6, 26, 27], with particular relevance within social settings.

In this context, the study of signed networks has benefited enormously from the quick growth of data on online social networks and more models are needed to understand its particular dynamics. In this work we report the results of a simulation approach to understand how the signs of the networks can evolve taking into account the social theories of structural balance and dynamics of peer-influence. We observed that updating a relation between two individuals based on the relations between both and the friends in common (triads) have impact in the evolution of the structural balance, but we also noticed that the way the signs of the network are initially attributed to each relation is determinant. The principles outlined in the proposed update rule for signs of links are general enough to be applied to other dynamical processes occurring in static and dynamic networks, where the sign (or weights) of ties plays an important role, from spreading of contagious diseases to diffusion of information in social networks.

For future work we plan to extend this study applying other sign distributions and update rules, including a probability approach similar to Antal [14] approach, but with the probabilities being proportional to the positive/negative counts explained in this work.

Acknowledgements This work was partly supported by national funds through Universidade de Lisboa and FCT—Fundação para a Ciência e Tecnologia, under projects TUBITAK/0004/2014, PTDC/EEI-SII/5081/2014, PTDC/EEI-SII/1973/2014, PTDC/MAT/STA/335 8/2014 and UID/CEC/50021/2013.

References

1. Harary, F.: On the notion of balance of a signed graph. Mich. Math. J. **2**, 143–146 (1954)
2. Cartwright, D., Harary, F.: Structural Balance: a Generalization of Heider's theory. Psychol. Rev. **63**, 277–292 (1956)
3. Davis, J.A.: Clustering and structural balance in graphs. Hum. Relat. **20**, 181–187 (1967)
4. Hummon, N.P., Doreian, P.: Some dynamics of social balance processes: bringing Heider back into balance theory. Soc. Netw. **25**(1), 17–49 (2003)
5. Leskovec, J., Huttenlocher, D., Kleinberg, J.: Signed networks in social media. In: 28th ACM Conference on Human Factors in Computing Systems (CHI) (2010)
6. Leskovec, J., Huttenlocher, D., Kleinberg, J.: Predicting positive and negative links in online social networks. In: 28th ACM Conference on Human Factors in Computing Systems (CHI) (2010)
7. Facchetti, G., Iacono, G., Altafini, C.: Computing global structural balance in large-scale signed social networks. Proc. Natl. Acad. Sci. USA **108**, 20953–20958 (2011)
8. Estrada, E., Benzi, M.: Are social networks really balanced? arXiv:1406.2132 [physics.soc-ph] (2014)
9. Skyrms, B.: The Stag Hunt and the Evolution of Social Structure. Cambridge University Press (2004)
10. Kossinets, G., Watts, D.J.: Empirical analysis of an evolving social network. Science **311**(5757), 88–90 (2006)
11. Santos, F.C., Pacheco, J.M., Lenaerts, T.: Cooperation prevails when individuals adjust their social ties. PLoS Comput. Biol. **2**(10), e140 (2006)
12. Gross, T., Blasius, B.: Adaptive coevolutionary networks: a review. J. R. Soc. Interface **5**(20), 259–271 (2008)
13. Barabási, A.-L.: Network Science. Cambridge University Press (2016)
14. Antal, T., Krapivsky, P.L., Redner, S.: Dynamics of social balance on networks. Phys. Rev. Lett. **72**, 036121 (2005)
15. van de Rijt, A.: The micro-macro link for the theory of structural balance. J. Math. Sociol. **35**, 94–113 (2011)
16. Dorogovtsev, S.N.: Lectures on Complex Networks. Oxford University Press, Oxford (2010)
17. Christakis, N.A., Fowler, J.H.: The spread of obesity in a large social network over 32 years. N. Engl. J. Med. **357**(4), 370–379 (2007)
18. Christakis, N.A., Fowler, J.H.: The collective dynamics of smoking in a large social network. N. Engl. J. Med. **358**(21), 2249–2258 (2008)
19. Pinheiro, F.L., Santos, M.D., Santos, F.C., Pacheco, J.M.: Origin of peer influence in social networks. Phys. Rev. Lett. **112**(9), 098702 (2014)
20. Pastor-Satorras, R., Castellano, C., Van Mieghem, P., Vespignani, A.: Epidemic processes in complex networks. Rev. Mod. Phys. **87**(3), 925 (2015)
21. Santos, F.C., Pacheco, J.M., Lenaerts, T.: Evolutionary dynamics of social dilemmas in structured heterogeneous populations. Proc. Natl. Acad. Sci. USA **103**(9), 3490–3494 (2006)
22. Santos, F.C., Santos, M.D., Pacheco, J.M.: Social diversity promotes the emergence of cooperation in public goods games. Nature **454**(7201), 213–216 (2008)
23. Szabó, G., Fath, G.: Evolutionary games on graphs. Phys. Rep. **446**(4), 97–216 (2007)
24. Rand, D.G., Nowak, M.A., Fowler, J.H., Christakis, N.A.: Static network structure can stabilize human cooperation. Proc. Natl. Acad. Sci. USA **111**(48), 17093–17098 (2014)
25. Castellano, C., Fortunato, S., Loreto, V.: Statistical physics of social dynamics. Rev. Mod. Phys. **81**(2), 591 (2009)
26. Barabasi, A.-L.: The origin of bursts and heavy tails in human dynamics. Nature **435**(7039), 207–211 (2005)
27. Onnela, J.-P., et al.: Structure and tie strengths in mobile communication networks. Proc. Natl. Acad. Sci. USA **104**(18), 7332–7336 (2007)