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# **Mathematical Biology**



# Evolutionary dynamics of collective index insurance

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Abstract Index-based insurances offer promising opportunities for climate-risk investments in developing countries. Indeed, contracts conditional on, e.g., weather or livestock indexes can be cheaper to set up than conventional indemnity-based insurances, while offering a safety net to vulnerable households, allowing them to eventually escape poverty traps. Moreover, transaction costs by insurance companies may be additionally reduced if contracts, instead of arranged with single households, are endorsed by collectives of households that bear the responsibility of managing the division of the insurance coverage by its members whenever the index is surpassed, allowing for additional flexibility in what concerns risk-sharing and also allowing insurance companies to avoid the costs associated with moral hazard. Here we resort to a population dynamics framework to investigate under which conditions household collectives may find collective index insurances attractive, when compared with individual index insurances. We assume risk sharing among the participants of each collective, and model collective action in terms of an N-person threshold game. Compared to less afford-

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able individual index insurances, we show how collective index insurances lead to a coordination problem in which the adoption of index insurances may become the optimal decision, spreading index insurance coverage to the entire population. We further investigate the role of risk-averse and risk-prone behaviors, as well as the role of partial correlation between insurance coverage and actual loss of crops, and in which way these affect the original coordination thresholds.

Keywords Index insurance  $\cdot$  Collective action  $\cdot$  Evolutionary game theory  $\cdot$  Non-linear returns

### **1** Introduction

For over a decade, weather index insurances (insurance policies based on the use of local data and statistical information, measured and gathered via local stations) have become available in the developing world, to defray possible damage on crops that may result from levels of drought above a certain threshold (Deng et al. 2007; Giné et al. 2007: Osgood et al. 2007: Clarke and Kalani 2011: Clarke 2011: Gaurav et al. 2011; Clarke et al. 2012a, b; De Bock and Gelade 2012; Hossain 2013; De Janvry et al. 2014; Dercon et al. 2014; Keswell and Carter 2014; Norton et al. 2014). Indeed, such insurance policies offer promising opportunities for climate-risk management in developing countries. Given that agriculture is an uncertain business in these regions, making households economically vulnerable, and given that indemnity-based crop insurances are usually unaffordable by these households (Hazell 1992; Hess et al. 2005), contracts conditional only on weather indices can be cheaper and still offer protection against certain types of weather events (Hess et al. 2005). These policies typically are offered to individual households, so we term them Individual Index Insurances (III), to distinguish from the Collective Index Insurances (CII) that have received a lot of attention more recently (De Bock and Gelade 2012; Hossain 2013; De Janvry et al. 2014; Dercon et al. 2014) and will be the focus of this paper.

The idea of **CII** is very simple: Similar to **III**, **CII** subjects the contracts to a technologically objective index, such as a weather variable, so that when this variable exceeds the contracted value, the insurance company partially compensates for the loss concomitant with the outcome expected from such a weather event. It is worth pointing out that the compensation will happen irrespective of any actual loss, and thus, unlike indemnity insurance—where the damages are assessed in loco by insurance officials—the index insurance is not necessarily perfectly correlated with a real loss, with the damages assessed in loco by insurance officials (the situation when a loss is not perfectly correlated with surpassing the index value contracted will be addressed in detail in the "Appendix"). Instead, coverage is automatic, which means that management costs by the insurance company can be significantly reduced—both in **III** and **CII**. The automatic nature of index insurances also allows insurance companies to avoid costs related to moral hazard associated, e.g., with resolution of potential con-

flicts of interest with clients. It also discourages perverse practices often encountered in conventional indemnity-based insurances. In **CII**, however, management costs by the insurance companies can be additionally reduced compared to **III**, since the contract is no longer made with each single household, but instead with a collective of households. The fact that, in **CII**, it is the collective that bears the responsibility of managing the division of the insurance coverage by its members whenever the index is surpassed, allowing an asymmetric distribution of the insurance coverage by its members, may render **CII** more attractive compared to **III**. Indeed, it is very likely that not all members of a collective will be equally affected when the index is surpassed.

However attractive this idea may be, experience shows that in Ethiopia, where **CII** was offered to Collectives, the levels of "spontaneous" adoption were very small, although training of Collectives' leaders in risk-sharing improved the situation significantly (Clarke and Kalani 2011; Dercon et al. 2014). Other factors, such as the difficulty in offering well-designed insurance policies, may have also contributed to the limited adoption rates (Clarke and Kalani 2011; Dercon et al. 2014). This said, it is noteworthy that, in some cases, coverage by **III** or **CII** would give poor farmers access to (otherwise inaccessible) credit/micro-credit (some lenders, such as banks, in some cases provide loans contingent on individuals buying also index insurance) (Barnett et al. 2005; Carter et al. 2007; Cole et al. 2013) which makes the investment in new technologies possible, thus increasing the production yield, and enabling farmers to escape so-called "poverty traps" (Barrett and McPeak 2006).

Besides **III** and **CII**, weather markets and weather derivatives also exist that allow companies of all sizes to hedge their cash-flow against "bad" weather (Alaton et al. 2002). However, the complexities inherent to a derivatives market (Alaton et al. 2002) may seem daunting when farmers' collectives or (often informal) small enterprises in the developing world are at stake (Hazell 1992; Osgood et al. 2007). In such cases, a more conventional approach may be more appropriate. Here, we shall follow (Clarke 2011) in translating our model parameters into an actuarial nomenclature, albeit closer to more conventional insurance policies.

Here we investigate the feasibility of groups to adhere to **CII** in the framework of evolutionary game theory, and investigate under which conditions household collectives may find **CII** attractive. We shall ignore indemnity insurance, but will assume that a (potentially small) fraction of farmers will be able to contract **III**, whenever **CII** is not realizable. Furthermore, we assume that the decision that a given Collective adopts **CII** is a group decision, modeling this collective action problem in terms of a *N*-person game. Thus, when a group decides to adopt **CII**, insurance coverage is shared among all members of the group, a situation that may be associated with informal risk-sharing practices commonly encountered (Clarke and Kalani 2011; Dixit et al. 2013).

## 2 Model

Let us consider a population of Z individuals (farmers, households) who set up groups (collectives) of size N, in which individuals may opt for a **CII**; whether the joint group decision will translate into an effective **CII** will be modeled in terms of a threshold

*N*-person game (details below) (Pacheco et al. 2009; Souza et al. 2009; Santos and Pacheco 2011). Each individual is capable of adopting one of two strategies: C (willing to contract a **CII** or **III**—if affordable) and D (Denier of insurance).

The insurance companies offer both III and CII. CII will be contracted (at a smaller collective cost, which translates into an individual cost  $c_C$  if at least M ( $1 \le M \le N$ ) individuals in the collective opt for them (that is, if the number  $n_C$  of C s satisfies M). Otherwise, only III will be available (at an individual cost  $c > c_C$ ), and  $n_C >$ they will be purchased by a fraction  $\gamma$  of the Cs in the population (those who can afford them). Finally, insurance companies are called to cover possible damage that may happen when a given (say weather) index surpasses a value I—we assume that this process occurs with a probability r (the risk, which will be covered by both **CII** and **III**). r Thus represents the probability that a full loss occurs, which coincides with the probability that the index I is surpassed. This is clearly a simplification, and we shall address the general model in which there is no perfect correlation between these events in the "Appendix". Thus, a **D** loses the entire endowment b upon a full Loss, but keeps the entire endowment under no Loss, given that she/he will never adhere to any form of index insurance. Moreover, often insurance models postulate a distribution of losses. In our case, such a distribution would require us to abandon the mean-field approximation that lies at the heart of the present model. Thus, the same Loss that accrues to each and every individual will correspond, in general, to the mean of a Loss distribution.

As is typical of index insurances, payment is contingent on a weather value surpassing a given index *I*, but coverage is not 100% effective: only a fraction  $\alpha$  of the *loss L* (here equal to the initial wealth or endowment *b* of each individual, which we assume results from, say, crop production) is actually covered by the insurance. In actuarial terms, and using as a reference the **III**, we may define  $m = \frac{c}{\alpha b}$ , as the ratio between the cost of **III** and the insurance coverage. In what concerns **III**, m > 1 reflects so-called (from the insurance company perspective) actuarially favorable scenarios [those actually adopted in most scenarios to date (Clarke 2011)], m = 1 actuarially fair scenarios and m < 1 actuarially unfair scenarios. Furthermore, we may define  $s_C = \frac{c_C}{c}$  (a real number smaller than 1) as the net reduction (saving) in the cost of moving from **III** to **CII**, such that the same concepts associated with *m* for **III** apply for  $m*s_C$  regarding **CII**.

#### 2.1 Payoffs

We first consider the basic problem, without risk sensitivity, and then modify to show how risk sensitivity makes the scheme feasible. In the absence of such sensitivity, the payoffs that accrue to each participant in the group are, in all cases, the sum of two contributions: One that occurs when the insurance is called to act (probability r) and the other when nothing happens (probability 1-r). We further assume (for simplicity) that all individuals get identical benefit *b* from whatever activity they perform.

Thus, the payoffs  $\Pi$  of what we designate by the **CII**-game, detailed in Table 1 for *D*s and Table 2 for *C*s, translate in the following expressions:

$$\Pi_D = (1 - r)b$$
  
$$\Pi_C(n_C) = \Theta(n_C - M) \left[ r\alpha b + (1 - r)b - c_C \right]$$

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State	Probability	Payoff of <b>D</b> s
(0)	1 - r	b
(I)	r	0

Table 1Expected payoffs of Ds in the CII-game

From the insurance company perspective, compensation based on an index insurance is triggered by the parameter r, the probability that a full loss occurs, which in this basic model coincides with the probability that the index I is surpassed (see the "Appendix" for an extended model). Thus, a D loses the entire endowment b upon a full Loss, but keeps the entire endowment under no Loss, given that she/he will never adhere to any form of index insurance

Table 2	Expected	payoffs	of Cs	in the	CII-game

State	Probability	Payoff of <i>C</i> s	Payoff of <i>C</i> s		
		$n_C \ge M$	$n_C < M$		
(0)	1 - r	$b - c_C$	$\gamma(b-c) + (1-\gamma)b$		
(I)	r	$\alpha b - c_C$	$\gamma(\alpha b - c)$		

Same notation as in Table 1. Similar to Ds, a C will lose entire endowment b upon a full Loss if not ensured, losing only part of it  $(1 - \alpha)b$  if insured; insurance policy can be of **III**-type, in which case a C entails a cost c, or of **CII**-type, in which case a C entails a cost  $c_C$ . Only a fraction  $\gamma$  of all Cs can afford to buy **III**, whereas **CII** are available to collectives in which more that a critical fraction M/N of them agrees to adhere to a **CII** 

$$+ [1 - \Theta(n_C - M)] [r \gamma \alpha b + (1 - r)b - \gamma c]$$
  
=  $\Theta(n_C - M)\Omega + [1 - \Theta(n_C - M)] \Delta + \Pi_D$ 

with  $\Omega = r\alpha b - c_C = c(\frac{r}{m} - s_C)$  and  $\Delta = \gamma(r\alpha b - c) = \gamma c(\frac{r}{m} - 1)$  real valued functions;  $\Theta(k)$  is the Heaviside function (that is,  $\Theta(k) = 1$  whenever  $k \ge 0$ , being zero otherwise). We are implicitly assuming that a Loss is a full Loss, thus no benefit remains after a Loss.

#### 2.2 Fitness and evolutionary dynamics

Now we can study the evolutionary dynamics of this population with players adopting two different strategies. When risk (r) is small, Ds will always gather a payoff that is larger than that of Cs. Consequently, they will dominate the evolutionary dynamics. For increasing risk, what it pays to do will depend on the parameters—N, M, r, m, c,  $s_C$  and  $\gamma$ .

Let us assume large populations (Z large) and describe the dynamics using the Replicator Equation (**RE**):

$$\dot{x} = x(1-x)(f_C(x) - f_D(x))$$

where x denotes the fraction of Cs in the population. We are implicitly adopting the well-mixed or mean-field approximation, in which all Cs and all Ds have the same

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(average) fitness, such that the evolutionary dynamics can be completely specified in terms of x. We assume that groups of size N are formed by selecting individuals independently and at random from the much larger total population, and compute the fitness of any C and D in the population as the average payoff that an individual employing this strategy obtains when participating in groups with all possible compositions of Cs and Ds. To this end, it is enough to specify how many, e.g., Cs are present in a group. If we designate this number by k, the fitness of any C and any D is given by averaging the payoff that a focal individual (C or D) gets when participating in groups encompassing all possible compositions (note that the focal individual is part of the group, which means that sampling extends to the remaining N - 1 individuals)

$$f_C = \sum_{k=0}^{N-1} {\binom{N-1}{k}} x^k (1-x)^{N-1-k} \Pi_C(k+1)$$
$$f_D = \sum_{k=0}^{N-1} {\binom{N-1}{k}} x^k (1-x)^{N-1-k} \Pi_D$$

so that fixed points occur at x = 0, x = 1, and possibly at intermediate points  $x^* \in (0, 1)$ , satisfying  $f_C(x^*) = f_D(x^*)$ . Using the expressions above for the payoffs, we may write:

$$f_C(x) - f_D(x) = \sum_{k=0}^{N-1} {\binom{N-1}{k} x^k (1-x)^{N-1-k} \left[ \prod_C (k+1) - \prod_D \right]}.$$

Let us define  $0 \le B_{N,M}(x) \equiv \sum_{k=M-1}^{N-1} \Phi(N-1,k,x) = \sum_{k=M-1}^{N-1} \binom{N-1}{k}$  $x^{k}(1-x)^{N-1-k} \leq 1.$ Given that  $\sum_{k=0}^{N-1} \Phi(N-1,k,x) = 1$ , we may write

$$f_C(x) - f_D(x) = \sum_{k=0}^{N-1} \Phi(N-1, k, x) \left[ \Pi_C(k+1) - \Pi_D \right]$$
  
= 
$$\sum_{k=0}^{N-1} \Phi(N-1, k, x) \left[ \Theta(k+1-M) \Omega + [1 - \Theta(k+1-M)] \Delta + \Pi_D - \Pi_D \right]$$
  
= 
$$\Delta + (\Omega - \Delta) B_{N,M}(x)$$

#### **3 Results**

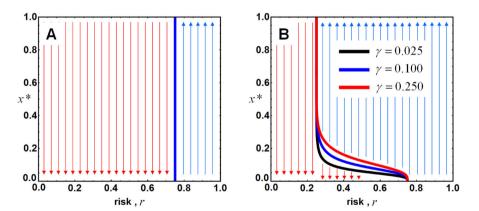
Since  $B_{N,M}(x)$  is a monotone (increasing) function of x,  $f_C(x) - f_D(x)$  will have at most one root  $x^*$  in the interval (0, 1). For  $0 < \gamma < 1$ ,  $x^*$  must satisfy the equation

$$B_{N,M}(x^*) = \frac{\Delta}{\Delta - \Omega} = \frac{1}{1 - \frac{1}{\gamma} \frac{r - ms_C}{r - m}}$$
(1)

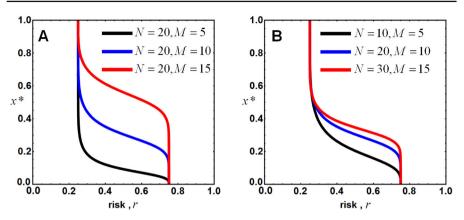
For an internal root to exist, we must have either  $\Delta > 0 \land \Omega < 0$  or  $\Delta < 0 \land \Omega > 0$ ; the first condition is impossible, whereas the second leads to  $ms_C < r < m$ . Moreover, whenever r > m, Cs will dominate unconditionally; whenever  $r < ms_C$ , Ds will dominate unconditionally. Whenever  $ms_C < r < m$ , the structure of  $f_C(x) - f_D(x)$ shows that, for small x,  $f_C(x) < f_D(x)$  whereas for large x,  $f_C(x) > f_D(x)$ ; this, in turn, implies that the internal fixed point of the **RE** is unstable—a coordination point. This said, Eq. (1) clearly shows that the detailed behavior of the coordination boundary is also determined by  $B_{N,M}(x)$ . The behavior of this monotonic function as a function of group size N and collective threshold M is illustrated below in Fig. 2.

In the absence of **CII**, the only limit that is retained is r > m, and the overall dynamics is characterized by an *all-or-nothing* scenario: For high risk (r > m) the fraction  $\gamma$  of the population that can afford **III** will contract them, whereas the remaining fraction of the population will continue without any index insurance. On the other hand, as the risk *r* falls below the critical value r = m, no-one will adhere to index insurance. This, in turn, puts a lot of pressure into the design and planning of index insurances, given that only in the actuarially unfavorable scenarios will there be individuals contracting **III**. With regard to existing attempts to implement **III** in developing countries, where m > 1, our model predicts that no adherence will take place, independent of *r*. The scenario is portrayed with a solid blue line in Fig. 1a, where we show the critical value of risk,  $r^* = m$ , that leads to an overturn of the overall evolutionary dynamics.

Introduction of **CII** (also assuming individuals are risk-averse, see Sect. 3.1), leads to considerably less grim prospects for index insurance, as shown in Fig. 1b. Indeed, and depending on the cost-effectiveness of **CII** compared to **III**, there are now 3 domains of risk (for finite  $\gamma$ ), 2 of which open the possibility for *C*s to dominate over



**Fig. 1** a In the absence of **CII**, a sufficiently high risk r (r = m = 0.75) is all that matters in driving the fraction  $\gamma$  of *C*s that can afford an **III** to actually contract it. Upward (*blue*) and downward (*red*) *arrows* indicate the direction of evolution under natural selection, which favors the extinction of *C*s (x = 0) whenever r < m and their fixation (x = 1) whenever r > m. **b** In the presence of a **CII**, now there appears a region of *r*-values ( $0.25 \le r \le 0.75$ ) in which *C*s engage in a coordination game with *D*s. The width of this region is controlled by  $s_C$  ( $s_C = 1/3$  in this case). In this region, the critical mass of *C*s that will render this strategy dominant in the population is indicated by the *solid lines*. The different *solid lines* correspond to different values of the fraction  $\gamma$  ( $\gamma = \{0.025, 0.1, 0.25\}$  drawn with a *black*, *blue* and *red line*, respectively) of *C*s in the population that can afford **III** if **CII** fails. Other parameters: c = 0.3, N = 20, M = 5



**Fig. 2** a Effect of threshold *M* for fixed group size N = 20. We let *M* take the values M = 5 (*black*) M = 10 (*blue*) and M = 15 (*red*), showing that, all else being equal, the role of increasing *M* is to render coordination more difficult for **C**s. **b** Effect of increasing group size *N* for fixed *M*/*N* ratio. We fix M/N = 0.5 and let the group size *N* take the values N = 10 (*black*) N = 20 (*blue*) and N = 30 (*red*). Increasing *N* for fixed *M*/*N* not only renders coordination more difficult for **C**s, but also sharpens the transition between the critical values r = m and  $r = ms_C$ . This last effect is similar to what was obtained in Fig. 1b by reducing  $\gamma$ . Other parameters: c = 0.3, m = 3/4,  $s_C = 1/3$ 

**D**s: unconditional dominance (r > m) or conditional dominance  $(ms_C < r < m)$ , that is, for each value of risk *r* in this interval, there exists a critical fraction  $x^*$  of *C*s in the population above which *C*s dominate unconditionally over *D*s. The critical fraction  $x^*$ , as shown with a solid black line in Fig. 1b, decreases monotonously with increasing risk *r*, as depicted. The slope of  $x^*$  as a function of *r* depends on  $\gamma$ : the larger the value of  $\gamma$ , the larger the slope of the transition into a *C*-dominance scenario.

In Fig. 2 we illustrate the role of group size N and threshold number M in the evolutionary dynamics of the **CII**-game. In Fig. 2a we fix the group size to N = 20 and change the threshold M required to contract **CII**. Increasing M renders coordination more difficult for Cs, that is, the critical mass of Cs present in the population to lead to an overall coordination into contracting **CII** also increases.

On the other hand, Fig. 2b shows that increasing group size at fixed M/N (i) increases the average fraction of Cs required to successfully coordination into CII and (ii) sharpens the transition from  $r = ms_C$  to r = m, similar to what we obtain when reducing the fraction  $\gamma$  of Cs who can afford to contract III (compare with Fig. 1b).

Note that, in all cases, the transition always takes place between the lines r = m and  $r = ms_C$ . As  $\gamma \to 0$ , the coordination point becomes more and more independent of r, and, as shown in Fig. 1b, coordination becomes easier until it disappears completely for  $\gamma = 0$ , in which case *C*s dominate for  $r > ms_C$  and go extinct whenever  $r < ms_C$ . This simple scenario, possibly realizable in the poorest places, positions **CII** as a very important alternative to **III**, as the critical risk is reduced by  $s_C$ .

In general, the thresholds obtained in this case also represent the thresholds for profitability for the insurance companies, even in the absence of other costs the companies have. Thus, in the absence of government subsidies, such a scheme will not work. However, what makes insurance work is that individuals cannot sustain catastrophic

losses, and thus exhibit a sensitivity to risk that companies can average over. We turn to this modification in the next section.

#### 3.1 Role of risk-aversion

In order to investigate the role of risk-aversion in our framework, we replace *payoffs* by *utilities* when computing the fitness of different strategies. We may adopt any of the following standard forms:

$$u(x) = 1 - e^{-x/R}$$
(2)

where 1/R measures the degree of risk-aversion (R being known as risk tolerance), as well as the alternative form

$$u(x) = \frac{1}{1 - \rho} x^{1 - \rho}$$
(3)

where  $\rho$  is the so-called Arrow–Pratt coefficient of relative risk aversion (Arrow 1963). Given the structure of the payoffs defined before,

$$\Pi_X(n_C) = r \times A_X(n_C) + (1 - r) \times B_X(n_C)$$

where  $A_X$  and  $B_X$  are directly extracted from Tables 1 and 2 and X stands both for C or D, we may now write

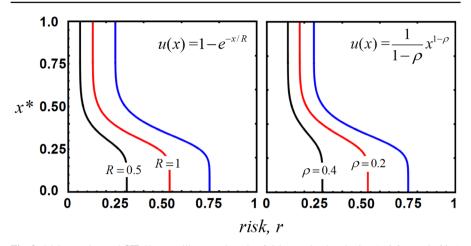
$$U_X(n_C) = r \times u \left[ A_X(n_C) \right] + (1 - r) \times u \left[ B_X(n_C) \right]$$

and proceed to compute the average fitness of Cs and Ds. Doing so we obtain, instead of Eq. (1) the following equation,

$$B_{N,M}(x^*) = \frac{rY_1 - (1-r)Y_2}{rY_3 - (1-r)Y_4}$$
(4)

where  $Y_k$ ,  $k \in \{1, 2, 3, 4\}$  read:  $Y_1 = (1 - e^{-\gamma(\alpha b - c)/R})$ ,  $Y_2 = (e^{-(b-\gamma c)/R} - e^{-b/R})$ ,  $Y_3 = (e^{-(\alpha b - c_C)/R} - e^{-\gamma(\alpha b - c)/R})$  and  $Y_4 = (e^{-(b-\gamma c)/R} - e^{-(b-c_C)/R})$ , corresponding to the utility definition in Eq. (2). For large values of the risk tolerance R, Eq. (4) we obtain the risk-neutral limit which leads to the same behavior as that discussed in the previous section, with the same asymptotic limits  $r = ms_C$  and r = m. The same happens when we employ Eq. (4) and make  $\rho = 0$ . For general values of R and  $\rho$ , it is no longer possible to obtain a corresponding explicit expression for the asymptotic behavior. Nonetheless, Fig. 3 illustrates what happens as we change the degree of risk-aversion: Overall, risk aversion favors coordination into a cooperative regime in which individuals end up contracting **CII**, which now takes place for smaller values of risk (black and red lines in Fig. 3), compared to the no risk-aversion limit (solid blue line in Fig. 3). Furthermore, closer inspection of Fig. 3 shows that increasing risk-aversion contributes to enlarging the overall area in the  $\{r, x^*\}$ -plane in which Cs dominate unconditionally, implying a widespread adherence to **CII**. This increase

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**Fig. 3** Risk-aversion and **CII**. Here we illustrate the role of risk-aversion by plotting (*solid curve* in *blue*) the same curve (with the same parameters) as in Fig. 2a, (N = 20, M = 10). Upon inclusion of risk aversion using the formulations of Eqs. (2) and (3), we obtain the curves plotted with *red* and *black lines*, for the different values of the risk tolerance R and relative risk aversion  $\rho$  indicated. Compared to the scenario without risk aversion portrayed, risk-aversion leads to more favorable scenarios for Cs, as the net qualitative effect of risk-aversion is to decrease, for the same value of risk r, the critical value of  $x^*$  above which Cs will adhere to CII, at the same time enlarging the interval of risk values for which Cs dominate unconditionally

takes place at the expense of both shrinkage of the area associated with D-dominance as well as of the area in which coordination is necessary.

#### **4** Discussion

The fact that **CII** allows insurance companies to save costs that can be reflected in the actual cost per individual ( $c_C$ ) that contracting a **CII** involves, compared to that of **III** (c), means that **CII** are more attractive than **III** in getting poor households insured, shifting the critical value of risk*r* above which *C*s may adhere to **CII** from r = m to  $r = ms_C$ .

On the other hand, the actuarial value of III, m, needs careful study and design, as the advantages of CII obtained here are always measured with respect to m. In this sense, m > 1 scenarios lead to the worst possible prospects for Cs to contract CII. Strictly speaking, the risk r is not independent of m, in the sense defined here, as r is the risk of surpassing a technological index I, ultimately defined in the CII contract. Thus, high values of I contribute to make r small, and this in turn may enable insurance companies to offer III with  $m \leq 1$ . In any case, taking into account the risk-averse nature of individuals renders possible situations in which insurance companies can profit from the CII policies provided. It is worth pointing out that climate change, by contributing to increase the occurrence of extreme events, also has a counter-productive effect in the design of index insurances (Vasconcelos et al. 2014). Furthermore, there is a delicate interplay between I, r and m, which is required to optimize index insurance policies. Finally, risk-premium valuation will be assessed

in different ways, depending on the types of indices chosen, and on whether they are traded or non-traded (Alaton et al. 2002). For instance, traded indices may pave the way to more complex strategies that we do not address here. This said, our parameter  $s_C$  will ultimately define the width of the windows, in risk space, in which a critical mass of *C*s is required before overall acceptance of **CII** is in place.

An important point in this context is related to the fact that an actual loss incurred by a household is not perfectly correlated with the act of surpassing the index Icontracted via **CII**. Indeed, *index insurances* are, at best, partially correlated with actual Loss: a harvest may be lost either by (i) local effects (detected or not) by (say) the nearest weather station (and hence not always covered by **III** and **CII**), or by (ii) other factors, such as pestilence, certainly not covered by **III** and **CII**. To take this feature into consideration, we follow (**Clarke 2011**) and define two probabilities: p, the probability that harvest is lost; q, the probability that any *index insurance* is activated (as defined previously already in terms of the risk r). In the "Appendix" we discuss in detail the effect of augmenting our model with these two probabilities. Counter-intuitively, we show that all that matters is still the probability q that *index insurance* is activated.

In summary, our model shows that **CII** offers a clear advantage over **III** in providing a viable option for individuals, more so if risk-averse, to adhere to this type of insurance policies, paving the way for a more resilient activity of farming in poor and emergent countries. Indeed, **CII** transforms scenarios in which an index insurance adoption is unlikely, into situations in which a critical mass of adherents is all that maybe required for **CII** to prevail in the population. Our model also shows in which way risk-aversion may act in favor of adhesion to **CII** which, in all cases, must be carefully designed to become a useful option for farmers whilst remaining profitable for the insurance companies. In this context, it would be interesting to investigate whether diversity in behavior towards risk-aversion will actually foster or inhibit overall adhesion to **CII**. Work along these lines is in progress.

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#### Compliance with ethical standards

Conflict of interest The authors declare no competing financial interests.

### Appendix: When actual loss and insurance coverage do not fully overlap

Here we follow (Clarke 2011) and investigate the effect of introducing two different probabilities instead of the risk r employed in the main text: p, the probability that

harvest is lost; q, the probability that any *index insurance* is activated (as defined previously already in terms of the risk r).

Depending on which one of the four possible states we are in, namely :

(0, 0)—(no Loss, no *insurance* activated) (0, *I*)—(no Loss, *insurance* activated) (*L*, 0)—(Loss, no *insurance* activated) (*L*, *I*)—(Loss, *insurance* activated)

we can write down the following payoff tables for *C*s and *D*s:

Ds			
State		Probability	Payoff
(0,0)		$p_{0,0}$	b
(0,I)		$p_{0,1}$	b
(L,0)		$P_{1,0}$	0
(L,I)		$p_{1,1}$	0
Cs State	Probability	Payoff	
State	FIODADIIIty	Fayon	
		$n_C \ge M$	$n_C < M$
(0,0)	<i>P</i> 0,0	$b - c_c$	$\gamma(b-c) + (1-\gamma)b$
(0, I)	$p_{0,1}$	$b(1+\alpha) - c_c$	$\gamma \left[ b(1+\alpha) - c \right] + (1-\gamma)b$
(L, 0)	$p_{1,0}$	$-c_c$	$\gamma(-c)$
(L, I)	$p_{1,1}$	$\alpha b - c_c$	$\gamma(\alpha b - c)$

In the simplest scenario, we may assume that *the probability p of occurrence of a loss is* statistically independent from *the probability q that the index I of the II is activated*. Then we can write for the probabilities of occurrence of each of the 4 states:

$$p_{0,0} = (1 - p) \times (1 - q)$$
  

$$p_{0,1} = (1 - p) \times q$$
  

$$p_{1,0} = p \times (1 - q)$$
  

$$p_{1,1} = p \times q$$

Such a statistical independence is unlikely, however. Thus, the joint probability distribution will not, in general, be the product of the 2 marginal probability distributions. Following (Clarke 2011), we investigate a symmetric joint probability distribution, defined in terms of a new parameter u, which corresponds to the probability that an individual will incur a Loss but without the *II* be activated ( $p_{1,0}$ ). In terms of u, the probabilities now read:

$$p_{0,0} = 1 - q - u$$
  $p_{0,1} = q - p + u$   
 $p_{1,0} = u$   $p_{1,1} = p - u$ 

Again, we can write for the fitness difference between *C*s and *D*s

$$f_C(x) - f_D(x) = \Delta + [\Omega - \Delta] B_{N,M}(x)$$

with  $\Delta = \gamma c \left(\frac{p_{0,1}+p_{1,1}}{m}-1\right)$  and  $\Omega - \Delta = c \left(\gamma + \frac{p_{0,1}+p_{1,1}}{m}-s_C\right)$ . The expressions for  $\Delta$  and  $\Omega - \Delta$  depend only on the combination  $p_{0,1} + p_{1,1} = q$ ,

The expressions for  $\Delta$  and  $\Omega - \Delta$  depend only on the combination  $p_{0,1} + p_{1,1} = q$ , despite the fact that, individually, both  $p_{0,1}$  and  $p_{1,1}$  depend on p, q and u. Since  $p_{0,1} + p_{1,1} = q$ , irrespective of whether the p and q distributions are independent or not, this means that the results we obtain for the original model also hold for this "extended model" provided that the original risk r is replaced by the probability q.

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